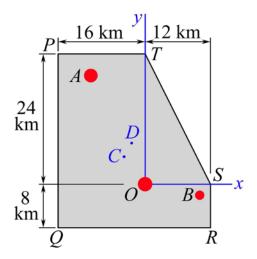**1.** ② Using the **principle of moments** (POM), describe (*a*) POM$_1$: the "resultant," (*b*) POM$_2$: the *moment* of the "resultant."

2. ⑧ Oshkosh, Akin, and Butler are major towns in the county shown, which have populations of 8000, 6000, and 2000, and locations at $O(0,0)$ km, $A(-10, 20)$ km, and $B(10, -2)$ km, respectively. For this county, locate (*a*) its geographic center $C(\overline{x}_G, \overline{y}_G)$, (*b*) its approximate population center $D(\overline{x}_P, \overline{y}_P)$.

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1. (*a*) POM$_1$: The "resultant" is equal to the *sum* of the "components." ①

   (*b*) POM$_2$: The *moment* of the "resultant" is equal to the *sum of the moments* of the "components," computed about the same point, axis, or plane. ①

2. We apply the POM *first* to areas and *then* to populations as follows:

Component areas: $A_1 = 16(24) = 384$, $A_2 = 28(8) = 224$, $A_3 = \frac{1}{2}(12)(24) = 144$

POM$_1$: Resultant area: $A = A_1 + A_2 + A_3 = 752$ ①

   Centroids: $C_1(-8, 12)$, $C_2(-2, -4)$, $C_3(4, 8)$, $C(\overline{x}_G, \overline{y}_G)$

POM$_2$: $\overline{x}_G(752) = -8(384) - 2(224) + 4(144)$ $\overline{x}_G = -3.9149$ ①

POM$_2$: $\overline{y}_G(752) = 12(384) - 4(224) + 8(144)$ $\overline{y}_G = 6.4681$ ①

   ∴   The geographic center is at $C(-3.91, 6.47)$ km. ①

   Component populations: $P_O = 8000$, $P_A = 6000$, $P_B = 2000$

POM$_1$: Resultant population $= P = P_O + P_A + P_B = 16000$ ①

   Locations: $O(0,0)$, $A(-10, 20)$, $B(10, -2)$, $D(\overline{x}_P, \overline{y}_P)$

POM$_2$: $\overline{x}_P(16000) = 0 - 10(6000) + 10(2000)$ $\overline{x}_P = -2.5$ ①

POM$_2$: $\overline{y}_P(16000) = 0 + 20(6000) - 2(2000)$ $\overline{y}_P = 7.25$ ①

   ∴   The population center is at $D(-2.5, 7.25)$ km. ①