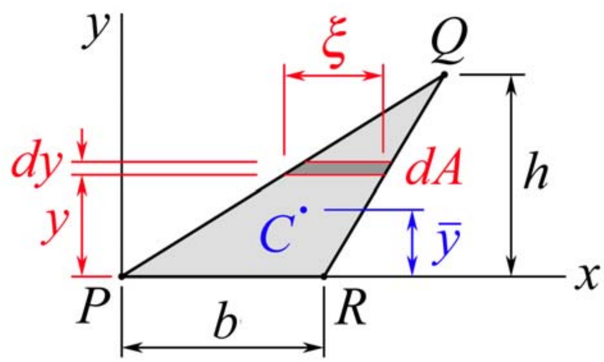
MEEG 2003 Quiz #6.m18

A triangular area PQR is shown, where C is its centroid and dA is its differential area. (a) Determine ξ, as indicated, and dA in terms of b, h, and y, (b) Derive its total area A and the ordinate $\bar{y}$ of C in terms of b and h using the principle of moments and calculus.



By similar triangles & treating dA as a rectangular area:

$$\frac{\xi}{b} = \frac{h-y}{h} \qquad \xi = \frac{b}{h}(h-y) \quad ② \qquad dA = \xi\, dy \qquad dA = \frac{b}{h}(h-y)\,dy \quad ②$$

By POM$_1$ and calculus:

$$A = \int dA = \int_0^h \frac{b}{h}(h-y)\,dy = \frac{b}{h}\left(hy - \tfrac{1}{2}y^2\right)\Big|_0^h = \frac{b}{h}\left(h^2 - \tfrac{1}{2}h^2\right) = \frac{bh}{2}$$

$$A = \tfrac{1}{2}bh \qquad ③$$

By POM$_2$ and calculus:

$$\bar{y}A = \int y\,dA = \int_0^h \frac{b}{h}(hy - y^2)\,dy = \frac{b}{h}\left(\tfrac{1}{2}hy^2 - \tfrac{1}{3}y^3\right)\Big|_0^h = \frac{bh^2}{6}$$

$$\bar{y} = \frac{1}{A}\cdot\frac{bh^2}{6} = \frac{2}{bh}\cdot\frac{bh^2}{6} = \frac{h}{3} \qquad \bar{y} = \frac{h}{3} \quad ③$$