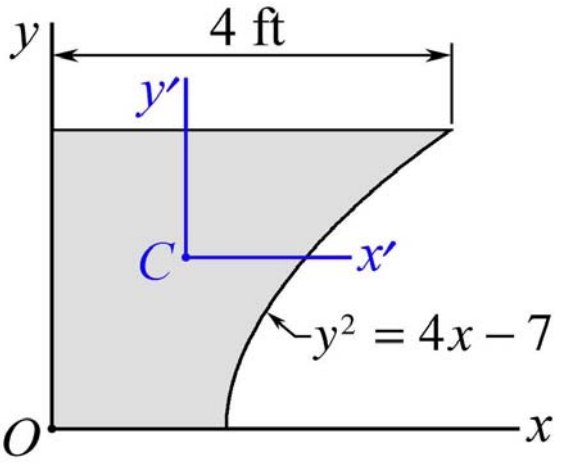
MEEG 2003 Quiz #7.m23.083

1. Including a *sketch*, describe the *parallel-axis theorem* for area moments of inertia. ②

2. A shaded area is shown, where $C(\bar{x},\bar{y})$ is its centroid. For this shaded area, determine *(a)* the value of $\bar{x}$, *(b)* the moment of inertia I_y, *(c)* the centroidal moment of inertia $\bar{I}_{y'}$. ⑧



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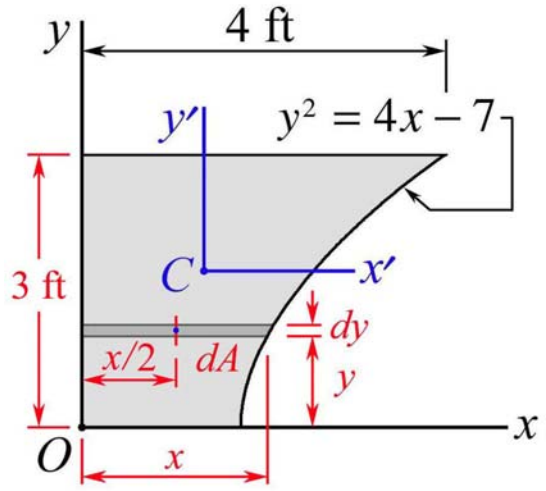
1. Description of PAT with a sketch ②

2.



①

$$x=\tfrac{1}{4}(7+y^2)$$

$$\bar{x}_{el}=\frac{x}{2}=\tfrac{1}{8}(7+y^2)$$

$$dA=x\,dy=\tfrac{1}{4}(7+y^2)\,dy$$

$$dI_y=\frac{1}{12}(dy)x^3+(x\,dy)\left(\frac{x}{2}\right)^2$$

$$=\frac{1}{3}x^3dy=\frac{1}{192}(7+y^2)^3dy \quad ①$$

POM$_1$: $A=\int dA=\int_0^3 \tfrac{1}{4}(7+y^2)dy=7.5$ $\quad A=7.5\ \text{ft}^2$ ①

POM$_2$: $\bar{x}A=\int \bar{x}_{el}\,dA=\frac{1}{32}\int_0^3(7+y^2)^2dy=10.05$ $\quad \bar{x}=1.34\ \text{ft}$ ①

$$I_y=\int dI_y=\int_0^3\tfrac{1}{192}(7+y^2)^3dy=\frac{2687}{140}=19.193 \quad I_y=19.19\ \text{ft}^4 \quad ②$$

PAT: $I_y=\bar{I}_{y'}+A\bar{x}^2$ $\quad \bar{I}_{y'}=5.7259$ $\quad \bar{I}_{y'}=5.73\ \text{ft}^4$ ②