MEEG 2013 Quiz #1.m03

The acceleration of a particle in rectilinear motion is given by $a = -\omega^2 x$, where ω is a constant. If $x_0 = 4$ m, $v_0 = 0$, and $v = -12$ m/s when $x = 0$, determine (*a*) the value of ω, (*b*) the time t elapsed during which the particle returns from $x = x_0 = 4$ m to $x = 0$.

$$a = v\frac{dv}{dx} = -\omega^2 x \qquad \int_0^v v\,dv = \int_4^x -\omega^2 x\,dx$$

$$\tfrac{1}{2}(v^2 - 0) = -\tfrac{1}{2}\omega^2 x^2\Big|_4^x = -\tfrac{1}{2}\omega^2(x^2 - 16)$$

$$v^2 = \omega^2(16 - x^2) \qquad (-12)^2 = \omega^2(16 - 0)$$

$$\omega^2 = 9 \qquad \omega = \pm 3\ \text{s}^{-1} \qquad ⑤$$

$$\frac{dx}{dt} = v = \pm 3(16 - x^2)^{1/2} \qquad \int_0^t \pm 3\,dt = \int_4^0 \frac{dx}{(16 - x^2)^{1/2}}$$

Letting $x = 4\sin\theta$, we write

$$\pm 3t = \int_{\pi/2}^0 \frac{4\cos\theta\,d\theta}{4\cos\theta} = -\frac{\pi}{2}$$

Since $t > 0$, we write

$$t = \left| -\frac{\pi}{2(\pm 3)} \right| = \frac{\pi}{6} = 0.5236 \qquad t = 0.524\ \text{s} \qquad ⑤$$