MEEG 2013 Quiz #1.m04.082

1. In curvilinear motion of particles, describe the magnitude and direction of (*a*) the normal component of acceleration $\mathbf{a}_n$, (*b*) the tangential component of acceleration $\mathbf{a}_t$. ②

2. Including sketches, derive the formulas for $\dot{\mathbf{e}}_r$ and $\dot{\mathbf{e}}_\theta$, which are the *time rates of change* of the unit vectors $\mathbf{e}_r$ and $\mathbf{e}_\theta$ in the polar coordinate system, respectively. ④

3. Making use of the formulas for $\dot{\mathbf{e}}_r$ and $\dot{\mathbf{e}}_\theta$ in Problem 2, derive in polar coordinates (*a*) the velocity $\mathbf{v}$, (*b*) the acceleration $\mathbf{a}$. ④

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1. (*a*) The magnitude of $\mathbf{a}_n$ equals the square of the speed v divided by the radius of curvature ρ of the path; i.e., $a_n = v^2/\rho$. The direction of $\mathbf{a}_n$ points from the particle toward the center of curvature of the path. ① (*b*) The magnitude of $\mathbf{a}_t$ equals the time derivative of the speed v; i.e., $a_t = dv/dt$. The direction of $\mathbf{a}_t$ is tangent to the path of the particle. ①

2. Sketches showing the unit vectors $\mathbf{e}_r$, $\mathbf{e}_\theta$, $\mathbf{e}'_r$, $\mathbf{e}'_\theta$, as well as $\Delta\mathbf{e}_r = \Delta\theta\,\mathbf{e}_\theta$ and $\Delta\mathbf{e}_\theta = -\Delta\theta\,\mathbf{e}_r$. ②

Use of calculus to show that $\dot{\mathbf{e}}_r = \dot{\theta}\,\mathbf{e}_\theta$ ①

Use of calculus to show that $\dot{\mathbf{e}}_\theta = -\dot{\theta}\,\mathbf{e}_r$ ①

3. Position vector of the particle: $\mathbf{r} = r\,\mathbf{e}_r$ ①

Use of the formulas $\dot{\mathbf{e}}_r = \dot{\theta}\,\mathbf{e}_\theta$, $\dot{\mathbf{e}}_\theta = -\dot{\theta}\,\mathbf{e}_r$ and the chain rule of differentiation to show that

$$\mathbf{v} = d\mathbf{r}/dt \qquad \mathbf{v} = \dot{r}\,\mathbf{e}_r + r\dot{\theta}\,\mathbf{e}_\theta \quad ①$$

$$\mathbf{a} = d\mathbf{v}/dt \qquad \mathbf{a} = (\ddot{r} - r\dot{\theta}^2)\,\mathbf{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\,\mathbf{e}_\theta \quad ②$$