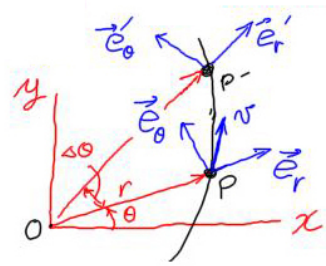
MEEG 2013 Quiz.172.m04

1. ⑥ Including sketches, derive the expressions of $\dot{\mathbf{e}}_r$ and $\dot{\mathbf{e}}_\theta$, which are the time rates of changes of the unit vectors $\mathbf{e}_r$ and $\mathbf{e}_\theta$.

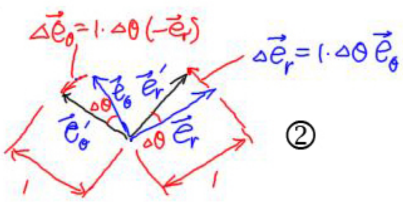
2. ④ Based on results obtained in Prob. 1, derive the *velocity* and *acceleration* in polar coordinates for a particle.

1.



②

$$\dot{r} = \frac{dr}{dt} \qquad \ddot{r} = \frac{d^2 r}{dt^2} \qquad \dot{\theta} = \frac{d\theta}{dt} \qquad \ddot{\theta} = \frac{d^2\theta}{dt^2}$$



②

$$\Delta\vec{e}_r = \Delta\theta\,\vec{e}_\theta \qquad \Delta\vec{e}_\theta = -\Delta\theta\,\vec{e}_r$$

$$\dot{\vec{e}}_r = \frac{d\vec{e}_r}{dt} = \lim_{\Delta t\to 0}\frac{\Delta\vec{e}_r}{\Delta t} = \lim_{\Delta t\to 0}\frac{\Delta\theta}{\Delta t}\vec{e}_\theta = \frac{d\theta}{dt}\vec{e}_\theta$$

$$\boxed{\dot{\vec{e}}_r = \dot{\theta}\,\vec{e}_\theta}$$ ①

①

$$\dot{\vec{e}}_\theta = \frac{d\vec{e}_\theta}{dt} = \lim_{\Delta t\to 0}\frac{\Delta\vec{e}_\theta}{\Delta t} = \lim_{\Delta t\to 0}\left(-\frac{\Delta\theta}{\Delta t}\vec{e}_r\right) = -\frac{d\theta}{dt}\vec{e}_r \qquad \boxed{\dot{\vec{e}}_\theta = -\dot{\theta}\,\vec{e}_r}$$

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2. $\vec{r} = r\,\vec{e}_r \qquad \vec{v} = \frac{d\vec{r}}{dt} = \dot{\vec{r}} = \dot{r}\vec{e}_r + r\dot{\vec{e}}_r = \dot{r}\vec{e}_r + r\dot{\theta}\,\vec{e}_\theta$

$$\boxed{\vec{v} = \dot{r}\,\vec{e}_r + r\dot{\theta}\,\vec{e}_\theta}$$ ②

$$\vec{a} = \frac{d\vec{v}}{dt} = \dot{\vec{v}} = \ddot{r}\vec{e}_r + \dot{r}\dot{\vec{e}}_r + \dot{r}\dot{\theta}\vec{e}_\theta + r\ddot{\theta}\vec{e}_\theta + r\dot{\theta}\dot{\vec{e}}_\theta$$

$$= \ddot{r}\vec{e}_r + \dot{r}\dot{\theta}\vec{e}_\theta + \dot{r}\dot{\theta}\vec{e}_\theta + r\ddot{\theta}\vec{e}_\theta + r\dot{\theta}(-\dot{\theta}\vec{e}_r)$$

$$= (\ddot{r} - r\dot{\theta}^2)\vec{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\vec{e}_\theta$$

$$\boxed{\vec{a} = (\ddot{r} - r\dot{\theta}^2)\vec{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\vec{e}_\theta}$$ ②