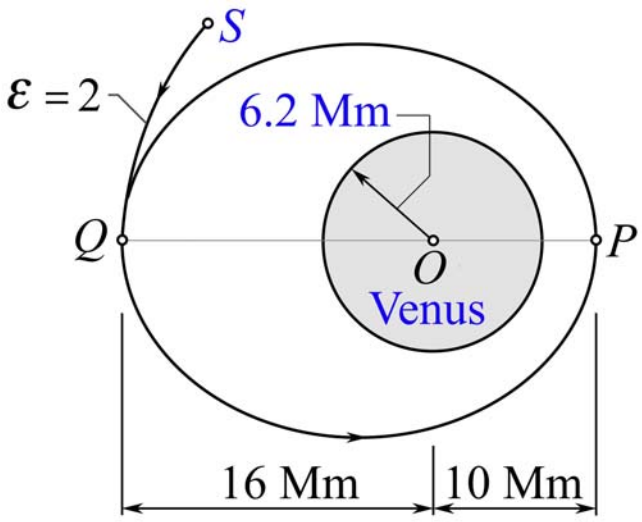
MEEG 2013 Quiz #2.m13

A spacecraft *S* approaches Venus along a hyperbolic trajectory *SQ* of eccentricity $\varepsilon = 2$ as shown. As it reaches *Q*, retrorockets are fired momentarily to insert it into an elliptic orbit as indicated. If the mass of Venus is 0.8144 times of the mass of the Earth, determine for the spacecraft (*a*) its speed as it approaches *Q*, (*b*) its speed after firing of retrorockets at *Q*, (*c*) the time required to travel from *Q* to *P*.



Let kilometer (km) and seconds (s) be used in the calculations.

(*a*) $\dfrac{1}{r_Q} = \dfrac{GM_V}{h_1^2}(1+\varepsilon\cos\theta) = \dfrac{GM_E\,(M_V/M_E)}{r_Q^2\,(v_Q)_{\text{hyp}}^2}(1+\varepsilon\cos\theta), \qquad GM_E = g_E R_E^2$

$$\frac{1}{16\times10^3} = \frac{(9.81\times10^{-3})(6370)^2(0.8144)}{(16\times10^3)^2(v_Q)_{\text{hyp}}^2}(1+2\cos 0)$$

$(v_Q)_{\text{hyp}}$ = 7.80 km/s ④

(*b*) $\dfrac{1}{r_P} + \dfrac{1}{r_Q} = \dfrac{2GM_V}{h_2^2} = \dfrac{2GM_E\,(M_V/M_E)}{r_Q^2\,(v_Q)_{\text{ell}}^2}$

$$\frac{1}{10\times10^3} + \frac{1}{16\times10^3} = \frac{2\left(9.81\times10^{-3}\right)(6370)^2(0.8144)}{(16\times10^3)^2(v_Q)_{\text{ell}}^2}$$

$(v_Q)_{\text{ell}}$ = 3.94785 km/s

$(v_Q)_{\text{ell}}$ = 3.95 km/s ③

(*c*) $t_{QP} = \dfrac{\tau}{2} = \dfrac{\pi(r_P+r_Q)\sqrt{r_P r_Q}}{2h_2} = \dfrac{\pi[(10+16)\times10^3]\sqrt{(10)(16)}\times10^3}{2(16\times10^3)(3.94785)}$ s = 8178.5 s

t_{QP} = 2.27 hours ③