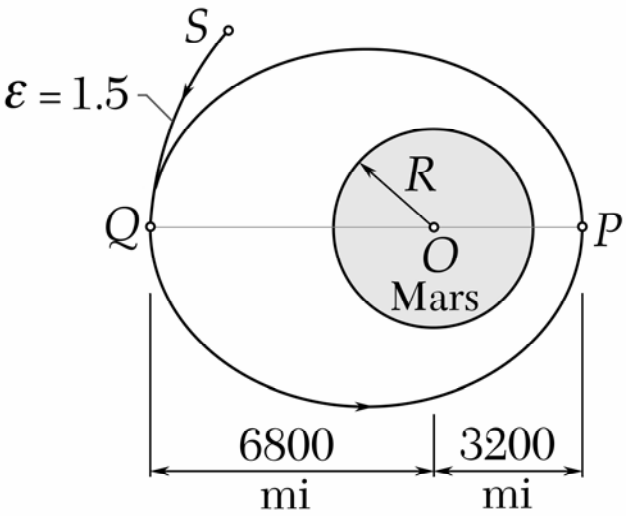
MEEG 2013 Quiz #2



A spacecraft approaches Mars along a hyperbolic trajectory *SQ* as shown. As it reaches *Q*, retrorockets are fired momentarily to insert it into an elliptic orbit as indicated. If the mass of Mars is 0.1077 times of the mass of the Earth, determine for the spacecraft (*a*) its speed as it approaches *Q*, (*b*) its speed after firing of retrorockets, (*c*) the time required to travel from *Q* to *P*.

$$(a)\quad \frac{1}{r_Q}=\frac{GM_M}{h_1^2}(1+\varepsilon\cos\theta)=\frac{GM_E\,(M_M/M_E)}{r_Q^2(v_Q)_{\text{hyp}}^2}(1+\varepsilon\cos\theta),\qquad GM_E=g_ER_E^2$$

$$\frac{1}{6800}=\frac{\frac{32.2}{5280}(3960)^2(0.1077)}{(6800)^2(v_Q)_{\text{hyp}}^2}(1+1.5\cos 0)$$

$(v_Q)_{\text{hyp}} = 1.946$ mi/s ◀

④

$$(b)\quad \frac{1}{r_P}+\frac{1}{r_Q}=\frac{2GM_M}{h_2^2}=\frac{2GM_E\,(M_M/M_E)}{r_Q^2(v_Q)_{\text{ell}}^2}$$

$$\frac{1}{3200}+\frac{1}{6800}=\frac{2\left(\frac{32.2}{5280}\right)(3960)^2(0.1077)}{(6800)^2(v_Q)_{\text{ell}}^2}$$

$(v_Q)_{\text{ell}} = 0.98458$ mi/s

$(v_Q)_{\text{ell}} = 0.985$ mi/s ◀

③

$$(c)\quad t_{QP}=\frac{\tau}{2}=\frac{\pi(r_P+r_Q)\sqrt{r_Pr_Q}}{2h_2}=\frac{\pi(3200+6800)\sqrt{3200(6800)}}{2(6800)(0.98458)}\text{ s}=10944\text{ s}$$

$t_{QP} = 3.04$ hours ◀

③