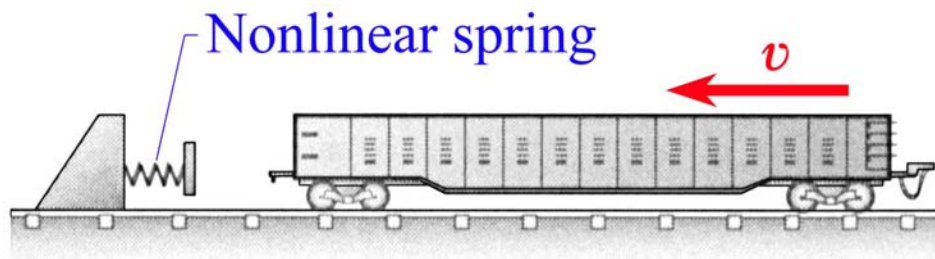


## MEEG 2013 Quiz #3.m18

A bumper made of a nonlinear spring is to stop a 30-ton gondola car that is traveling at a speed of  $v = 5$  ft/s as shown. The magnitude of the restoring force developed in the spring is  $F = 100x + 12x^3$  kips when it is compressed by an amount of  $x$  ft. Determine the maximum deflection  $\delta$  of the bumper.



$$T_1 + U_{1 \rightarrow 2} = T_2:$$

$$T_1 = \frac{1}{2}mv^2 = \frac{1}{2} \cdot \frac{30(2000)}{32.2} \cdot (5)^2 \text{ lb}\cdot\text{ft} = 23.2919 \times 10^3 \text{ lb}\cdot\text{ft} \quad \textcircled{2}$$

$$U_{1 \rightarrow 2} = -10^3 \cdot \int_0^{\delta} (100x + 12x^3) dx \text{ lb}\cdot\text{ft} \quad \textcircled{3}$$
$$= -10^3 \cdot (50x^2 + 3x^4) \Big|_0^{\delta} \text{ lb}\cdot\text{ft} = -10^3(50\delta^2 + 3\delta^4) \text{ lb}\cdot\text{ft}$$

$$T_2 = 0 \quad \textcircled{1}$$

$$23.2919 \times 10^3 - 10^3(50\delta^2 + 3\delta^4) = 0$$

$$3\delta^4 + 50\delta^2 - 23.2919 = 0 \quad \textcircled{2}$$

$$\delta^2 = 0.4535 \quad \text{or} \quad -17.1202 \text{ (discard)}$$

$$\delta = 0.6734 \text{ ft} \quad \therefore \delta = 8.08 \text{ in.} \quad \textcircled{2}$$