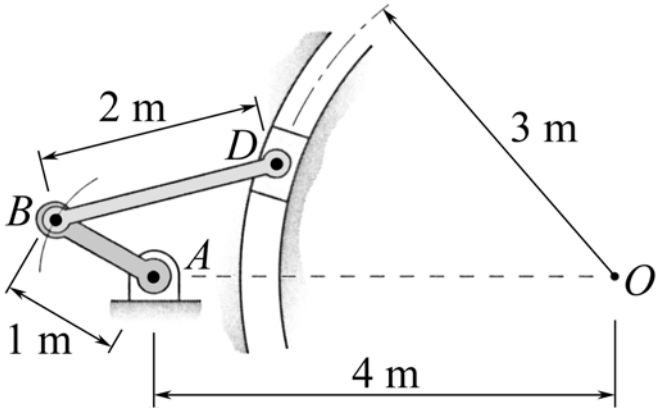
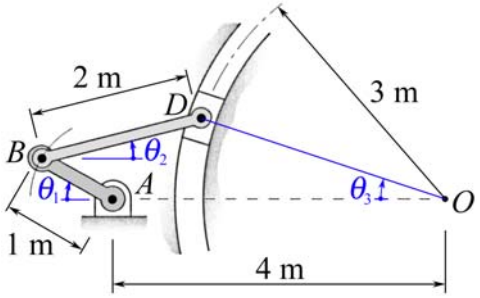
MEEG 2013 Quiz #5.m27

The crank *AB* rotates with a constant angular velocity $\boldsymbol{\omega}_{AB} = 2$ rad/s ↺ and the slider *D* moves in a circular groove. Determine the angular velocity $\boldsymbol{\omega}_{BD}$ of link *BD* and the velocity $\mathbf{v}_D$ of slider *D* when it *moves downward* and crosses the line *AO*.





①

The constraint condition is $\overrightarrow{AB} + \overrightarrow{BD} + \overrightarrow{DO} = \overrightarrow{AO}$. We write

$$1(-\cos\theta_1\mathbf{i} + \sin\theta_1\mathbf{j}) + 2(\cos\theta_2\mathbf{i} + \sin\theta_2\mathbf{j}) + 3(\cos\theta_3\mathbf{i} - \sin\theta_3\mathbf{j}) = 4\mathbf{i} \quad ①$$

$$-\cos\theta_1 + 2\cos\theta_2 + 3\cos\theta_3 = 4 \qquad (1) \quad ①$$

$$\sin\theta_1 + 2\sin\theta_2 - 3\sin\theta_3 = 0 \qquad (2) \quad ①$$

$$-(1 - \tfrac{1}{2}\theta_1^2) + 2(1 - \tfrac{1}{2}\theta_2^2) + 3(1 - \tfrac{1}{2}\theta_3^2) = 4 \qquad (1') \quad ①$$

$$\theta_1 + 2\theta_2 - 3\theta_3 = 0 \qquad (2') \quad ①$$

$$\theta_1^2 - 2\theta_2^2 - 3\theta_3^2 = 0 \qquad (1'')$$

$$\theta_3 = \tfrac{1}{3}(\theta_1 + 2\theta_2) \qquad (2'')$$

$$\theta_2 = 0.2\left(-1 \pm \sqrt{6}\right)\theta_1 \qquad \dot{\theta}_2 = 0.2\left(-1 \pm \sqrt{6}\right)\dot{\theta}_1 \qquad \dot{\theta}_3 = \tfrac{1}{3}(\dot{\theta}_1 + 2\dot{\theta}_2)$$

Since $\boldsymbol{\omega}_{AB} = 2$ rad/s ↺ and $\dot{\theta}_2$ must be negative, we write $\dot{\theta}_1 = -2$ rad/s and $\dot{\theta}_2 = -0.5798$ rad/s, $\dot{\theta}_3 = -1.0532$ rad/s, $v_D = \overline{OD}\,\dot{\theta}_3 = -3.1596$ m/s. Thus,

$\boldsymbol{\omega}_{BD} = 0.580$ rad/s ↻ ② $\quad$ $\mathbf{v}_D = 3.16$ m/s ↓ ②