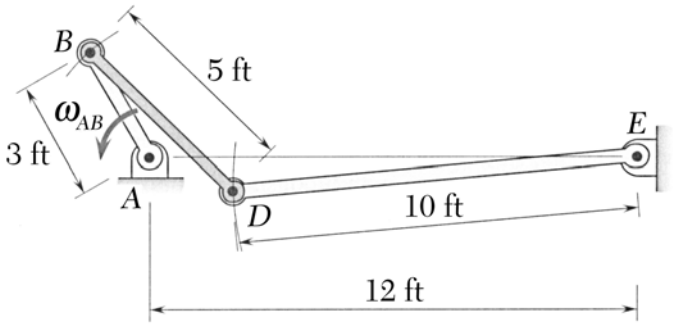
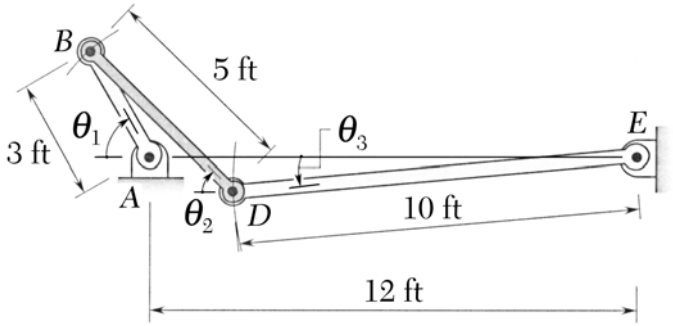
MEEG 2013 Quiz #5

The crank AB of a four-bar linkage rotates with a constant angular velocity $\boldsymbol{\omega}_{AB}$ as shown. It is known that the angular velocity of the output link DE is $\boldsymbol{\omega}_{DE}$ = 1 rad/s ↻ when AB, BD, and DE are collinear with the ground link AE. Determine the value of $\boldsymbol{\omega}_{AB}$.





Since $\overrightarrow{AB}+\overrightarrow{BD}+\overrightarrow{DE}=\overrightarrow{AE}$, we write

$$3(-\cos\theta_1\mathbf{i}+\sin\theta_1\mathbf{j})+5(\cos\theta_2\mathbf{i}-\sin\theta_2\mathbf{j})+10(\cos\theta_3\mathbf{i}+\sin\theta_3\mathbf{j})=12\mathbf{i}$$

$$-3\cos\theta_1+5\cos\theta_2+10\cos\theta_3=12 \qquad (1)$$

$$3\sin\theta_1-5\sin\theta_2+10\sin\theta_3=0 \qquad (2)$$

$$-3(1-\tfrac{1}{2}\theta_1^2)+5(1-\tfrac{1}{2}\theta_2^2)+10(1-\tfrac{1}{2}\theta_3^2)=12 \qquad (1')$$

$$3\theta_1-5\theta_2+10\theta_3=0 \qquad (2')$$

Equations (1′) and (2′) yield

$$3\theta_1^2-5\theta_2^2-10\theta_3^2=0 \qquad (1'')$$

$$\theta_2=0.6\theta_1+2\theta_3 \qquad (2'')$$

Substituting Eq. (2″) into Eq. (1″), we obtain

$$1.2\,\theta_1^2-12\,\theta_3\theta_1-30\theta_3^2=0 \qquad \text{or} \qquad \theta_1^2-10\theta_3\theta_1-25\,\theta_3^2=0$$

$$\theta_1=5(1\pm\sqrt{2})\theta_3 \qquad \therefore \quad \dot{\theta}_1=5(1\pm\sqrt{2})\dot{\theta}_3$$

Since $\boldsymbol{\omega}_{DE}$ = 1 rad/s ↻, we set $\dot{\theta}_3=-1$ rad/s to get $\dot{\theta}_1=-12.07$ rad/s.

$\therefore$ $\boldsymbol{\omega}_{AB}$ = 12.07 rad/s ↺ ◀