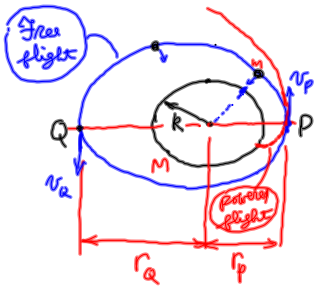
Central-force motion



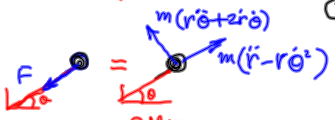
On the surface of the earth: $g = 9.81\ m/s^2$
$g = 32.2\ ft/s^2$

High above the surface of the earth:
$g < 9.81\ m/s^2 \quad g < 32.2\ ft/s^2$

Orbit: a closed trajectory

τ: period of orbit

Central force F:



$$F = \frac{GMm}{r^2} = GMm\,r^{-2}$$

$$\Sigma F_r: \quad -F = m(\ddot{r} - r\dot{\theta}^2)$$

$$\Sigma F_\theta: \quad 0 = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$$

$$\left.\begin{aligned} \cancel{m}(\ddot{r} - r\dot{\theta}^2) &= -GM\cancel{m}\,r^{-2} \\ \cancel{m}(r\ddot{\theta} + 2\dot{r}\dot{\theta}) &= 0 \end{aligned}\right\} \qquad \left.\begin{aligned} \ddot{r} - r\dot{\theta}^2 &= -GMr^{-2} \\ r\ddot{\theta} + 2\dot{r}\dot{\theta} &= 0 \end{aligned}\right\} \begin{aligned} &\cdots\cdots (1) \\ &\cdots\cdots (2) \end{aligned}$$

$$\frac{d}{dt}(r^2\dot{\theta}) = 2r\dot{r}\dot{\theta} + r^2\ddot{\theta} = r(r\ddot{\theta} + 2\dot{r}\dot{\theta}) = r(0) = 0$$

$\boxed{r^2\dot{\theta} = h}$ (a constant) $\qquad \vec{v} = \dot{r}\vec{e}_r + r\dot{\theta}\vec{e}_\theta = v_r\vec{e}_r + v_\theta\vec{e}_\theta$

$v_\theta = r\dot{\theta} \qquad h = r(r\dot{\theta}) = r v_\theta \qquad \boxed{r v_\theta = h}$ V.V.···I.

$$\boxed{\frac{1}{r} = \frac{GM}{h^2}(1 + \varepsilon\cos\theta)}$$

at P: $v_\theta = v_P,\ r = r_P$
at Q: $v_\theta = v_Q,\ r = r_Q$

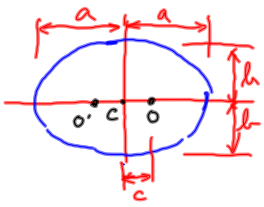
ε: eccentricity of the conic section

$\varepsilon = 0$: circle
$\varepsilon < 1$: ellipse
$\varepsilon = 1$: parabola
$\varepsilon > 1$: hyperbola

V.V.···I.

$$\boxed{\frac{1}{r_P} + \frac{1}{r_Q} = \frac{2GM}{h^2}} \qquad \boxed{\varepsilon = \frac{c}{a}}$$

$$\boxed{\tau = \frac{\pi(r_P + r_Q)\sqrt{r_P r_Q}}{h}}$$ V.V.···I.

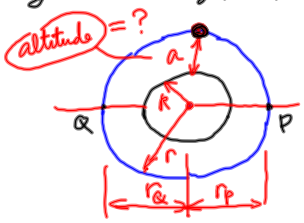


$$W = \cancel{m}g = \frac{GM\cancel{m}}{R^2}$$

$$\boxed{GM = gR^2}$$

$R = 3960\ mi \qquad R = 6370\ km = 6.37\ Mm$
$R = 3.96 \times 10^3\ mi \qquad R = 6.37 \times 10^3\ km$

Geostationary satellite: $r_P = r_Q = r \qquad \tau = \frac{\pi(r+r)\sqrt{rr}}{h} = \frac{\pi(2r^2)}{h} = \frac{2\pi r^2}{h}$



$$h = \frac{2\pi r^2}{\tau}$$

$$\frac{1}{r} + \frac{1}{r} = \frac{2GM}{h^2} \qquad \frac{2}{r} = \frac{2GM}{h^2}$$

$$h^2 = GMr = \frac{4\pi^2 r^4}{\tau^2}$$

$$GM\tau^2 = 4\pi^2 r^3 \qquad gR^2\tau^2 = 4\pi^2 r^3$$

$\tau = 24\,h = 24(3600)\,s$ ② ④

$$32.2(3960)^2(5280)^2(24)^2(3600)^2 = 4\pi^2 r^3$$

$$r = 1.3859 \times 10^8\ ft = 26247.8\ mi = R + a = 3960 + a$$

$$a = 22287\ mi \qquad \boxed{a = 22.3 \times 10^3\ mi}$$
