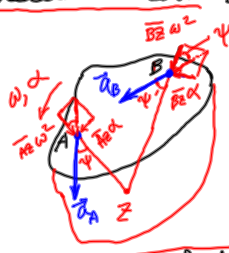
Acceleration center Z

$\vec{a}_Z = \vec{0}$ Z is a point of the body, or of the massless extension of the body.



$\vec{a}_A = \vec{a}_{A/Z} + \cancel{\vec{a}_Z} = \vec{a}_{A/Z}$

$\vec{a}_B = \vec{a}_{B/Z} + \cancel{\vec{a}_Z} = a_{B/Z}$

$\tan\psi = \dfrac{\overline{AZ}\,\alpha}{\overline{AZ}\,\omega^2} = \dfrac{\alpha}{\omega^2}$ $\quad \psi = \tan^{-1}\left(\dfrac{\alpha}{\omega^2}\right)$

$\tan\psi' = \dfrac{\overline{BZ}\,\alpha}{\overline{BZ}\,\omega^2} = \dfrac{\alpha}{\omega^2}$ $\quad \psi' = \tan^{-1}\left(\dfrac{\alpha}{\omega^2}\right)$

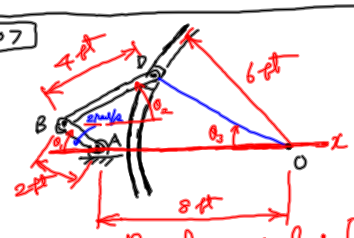
$\boxed{\psi' = \psi}$ If $\omega = 0$, then $\boxed{\psi = \psi' = 90^\circ}$

If a body is accelerated from rest, then $\omega = 0$ and $\psi = \psi' = 90^\circ$

Use acceleration center if $\omega = 0$.

If $\omega \neq 0$, then use linkage equation for accelerations to solve the problem.

15.107



$\vec{\omega}_{AB} = 2\ \text{rad/s}$ ↺

$\vec{\omega}_{BD} = ?$ $\quad \vec{v}_D = ?$

Four-bar linkage is mechanism.

Let θ_1, θ_2, & θ_3 be the parameters of the system. Note that

By polygon rule: $\boxed{\vec{AB} + \vec{BD} + \vec{DO} = 8\,\vec{i}}$

the constraint condition

$$2(-\cos\theta_1\,\vec{i} + \sin\theta_1\,\vec{j}) + 4(\cos\theta_2\,\vec{i} + \sin\theta_2\,\vec{j}) + 6(+\cos\theta_3\,\vec{i} - \sin\theta_3\,\vec{j}) = 8\,\vec{i}$$

$\vec{i}$: $-2\cos\theta_1 + 4\cos\theta_2 + 6\cos\theta_3 = 8$

$\vec{j}$: $2\sin\theta_1 + 4\sin\theta_2 - 6\sin\theta_3 = 0$

θ_1, θ_2, & θ_3 are infinitesimally small.

$\dot{\theta}_1 = -2\ \text{rad/s}$

$\sin\theta = \theta - \dfrac{\theta^3}{3!} + \dfrac{\theta^5}{5!} - \cdots \approx \theta$

$\cos\theta = 1 - \dfrac{\theta^2}{2!} + \dfrac{\theta^4}{4!} + \cdots \approx 1 - \frac{1}{2}\theta^2$

$-2(1 - \frac{1}{2}\theta_1^2) + 4(1 - \frac{1}{2}\theta_2^2) + 6(1 - \frac{1}{2}\theta_3^2) = 8$

$(\theta_1^2 - 2\theta_2^2 - 3\theta_3^2)\ \cancel{-2+4+6} = \cancel{8}$ $\quad \therefore \boxed{\theta_1^2 - 2\theta_2^2 - 3\theta_3^2 = 0}$

$\boxed{2\theta_1 + 4\theta_2 - 6\theta_3 = 0}$ $\qquad \theta_3 = \frac{1}{3}(\theta_1 + 2\theta_2)$

$\theta_1^2 - 2\theta_2^2 - \dfrac{3}{9}(\theta_1^2 + 4\theta_1\theta_2 + 4\theta_2^2) = 0$

$9\theta_1^2 - 18\theta_2^2 - 3\theta_1^2 - 12\theta_1\theta_2 - 12\theta_2^2 = 0$

$-30\theta_2^2 - 12\theta_1\theta_2 + 6\theta_1^2 = 0, \quad 5\theta_2^2 + 2\theta_1\theta_2 - \theta_1^2 = 0$

$\theta_2 = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{-2\theta_1 \pm \sqrt{4\theta_1^2 + 20\theta_1^2}}{10}$

$= \dfrac{-2\theta_1 \pm 2\sqrt{6}\,\theta_1}{10} = \frac{1}{5}(-\theta_1 \pm \sqrt{6}\,\theta_1)$

$\boxed{\theta_2 = \frac{1}{5}(-1 \pm \sqrt{6})\theta_1}$ $\qquad \boxed{\theta_3 = \frac{1}{3}(\theta_1 + 2\theta_2) = \square\,\theta_1}$

$\dot{\theta}_2 = \frac{1}{5}(-1 \pm \sqrt{6})(-2)$ $\qquad \dot{\theta}_3 = \frac{1}{3}(\dot{\theta}_1 + 2\dot{\theta}_2)$

$\dot{\theta}_1 = -2$, $\boxed{\dot{\theta}_2 = \frac{1}{5}(-1 + \sqrt{6})(-2), \quad \dot{\theta}_3 = \frac{1}{3}(\dot{\theta}_1 + 2\dot{\theta}_2)}$

$\dot{\theta}_1 = -2$ $\boxed{\dot{\theta}_2 = \frac{1}{5}(-1 - \sqrt{6})(-2), \quad \dot{\theta}_3 = \frac{1}{3}(\dot{\theta}_1 + 2\dot{\theta}_2)}$

$\omega_{BD} = \dot{\theta}_2$ $\qquad \boxed{\vec{\omega}_{BD} = \dot{\theta}_2\ ↺ \text{ if } \dot{\theta}_2 \text{ is positive}}$

$v_D = 6\dot{\theta}_3$ $\qquad \boxed{\vec{v}_D = 6\dot{\theta}_3\ ↑ \text{ if } \dot{\theta}_3 \text{ is positive}}$