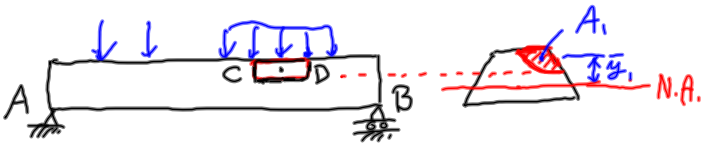
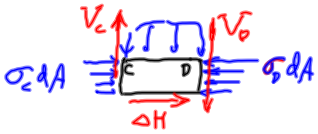
Longitudinal shear on a beam element of arbitrary shape



$$\sigma = \frac{My}{I}$$



$$\rightarrow \Sigma F_x = 0: \quad \int \sigma_c \, dA - \int \sigma_D \, dA + \Delta H = 0$$

$$\Delta H = \int \sigma_D \, dA - \int \sigma_c \, dA = \int (\sigma_D - \sigma_c) \, dA$$

$$\Delta H = \int \left(\frac{M_D y}{I} - \frac{M_c y}{I} \right) dA = \frac{M_D - M_c}{I} \int y \, dA = \frac{\Delta M}{I} Q$$

$$= \frac{\Delta M}{\Delta x} \frac{Q}{I} \Delta x \qquad \Delta H = \frac{dM}{dx} \frac{Q}{I} \Delta x = V \frac{Q}{I} \Delta x$$

$$q = \frac{\Delta H}{\Delta x} = \frac{VQ}{I} \qquad \boxed{q = \frac{VQ}{I}} \quad V, Q, \ldots I.$$

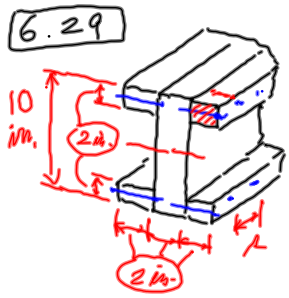
q: average longitudinal shear force per unit length of beam

Q: first moment of the area between the line of (clean) cut and the outer fibers $\quad (Q = \bar{y}_1 A_1)$

I: moment of inertia of the entire area about the neutral axis

V: vertical shear in the beam at C

6.29



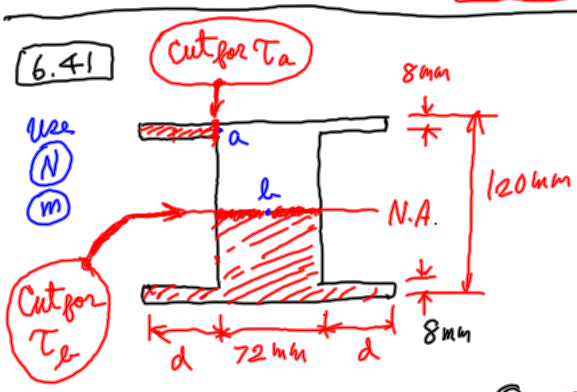
$V = 1200$ lb $\quad F_s = 75$ lb $\quad s = ?$

$$F_s = q s = \frac{VQs}{I}$$

$$75 = \frac{1200(5-1)(2)(2)s}{\frac{1}{12}(6)(10)^3 - \frac{1}{12}(2)(6)^3(2)} \qquad \therefore s = \square$$

$$\boxed{s = \square \text{ in.}}$$

6.41



$V = 25$ kN $\quad d = 50$ mm

$\tau_a = ?$ $\quad \tau_b = ?$

$$\tau = \frac{q}{t} = \frac{VQ}{It}$$

To find τ_a:

$V = 25 \times 10^3$ N, $\quad t = 0.008$ m

$Q = 0.056(0.05)(0.008)$ m^3

$I = \square$ m^4 $\qquad \therefore \tau_a = \square$ N/m^2

To find τ_b: Make a cut as shown.