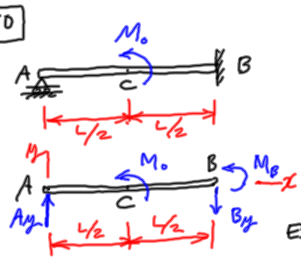
9.50




$\vec{A} = ?$ $\quad y_c = ?$

$q = A_y\langle x\rangle^{-1} - M_o\langle x - \frac{L}{2}\rangle^{-2}$

$V = A_y\langle x\rangle^{0} - M_o\langle x - \frac{L}{2}\rangle^{-1}$

$EIy'' = M = A_y\langle x\rangle^{1} - M_o\langle x - \frac{L}{2}\rangle^{0}$

$EIy' = \frac{A_y}{2}\langle x\rangle^{2} - M_o\langle x - \frac{L}{2}\rangle^{1} + C_1$

$EIy = \frac{A_y}{6}\langle x\rangle^{3} - \frac{M_o}{2}\langle x - \frac{L}{2}\rangle^{2} + C_1x + C_2$

Unknowns: $A_y$, $C_1$, & $C_2$

B.C. ① $y(0) = 0$: .....
② $y(L) = 0$: ....
③ $y'(L) = 0$: .....

$C_2 = \square$, $\quad C_1 = \square$, $\quad A_y = \square$

$\boxed{\vec{A} = \vec{A}_y = \square \uparrow}$ $\quad \boxed{y_c = y(\frac{L}{2}) = \square}$

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Method using moment-area theorems

1st moment-area theorem: $\boxed{\theta_{D/C} = A_{CD}}$ $\quad \theta_{D/C} = \theta_D - \theta_C$

Note that the point D of the beam must be on the right side of the beam, and $A_{CD}$ is the algebraic value of the elastic weight diagram between points C & D.

2nd moment-area theorem: $\boxed{t_{D/C} = (M_D)_{CD}}$

Note that $t_{D/C}$ is the tangential deviation of point D with respect to the tangent drawn at point C.

$t_{D/C}$ = vertical displacement from point D of the deflected beam to the tangent drawn at point C.

The sign of $(M_D)_{CD}$ must be consistent with the direction of the moment of $\vec{t}_{D/C}$ with respect to C

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Elastic weight $= \frac{M}{EI}$

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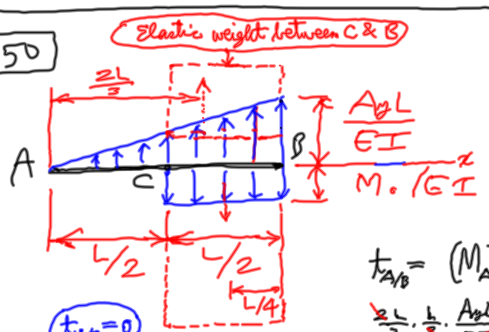
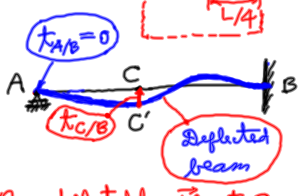



$\frac{M}{EI}$ diagram drawn by parts




Beam deflected by $\vec{M}_o$ at C

$t_{A/B} = (M_A)_{AB} = 0$:

$\frac{\cancel{2}L}{3} \cdot \frac{L}{\cancel{2}} \cdot \frac{A_yL}{\cancel{EI}} - \frac{3L}{4} \cdot \frac{M_o}{\cancel{EI}} \cdot \frac{L}{2} = 0$

$\frac{L^3}{3}A_y - \frac{3L^2}{8}M_o = 0$

$A_y = \frac{3}{L^3} \cdot \frac{3L^2}{8} M_o = \frac{9M_o}{8L}$

$\therefore \boxed{\vec{A} = \frac{9M_o}{8L} \uparrow}$

We may assume that the shape of the deflected beam is sketched as shown. Under such an assumption, note that $\vec{t}_{C/B} = \overrightarrow{C'C} \uparrow$. The moment of $\vec{t}_{C/B}$ with respect to the reference point B, where the tangent is drawn, is CLOCKWISE. Therefore, by the second moment-area theorem, we write

$\boxed{t_{C/B} = +2\,(M_C)_{CB}}$ = moment, about point C, of the elastic weight between points C and B.

$$t_{C/B} = -\frac{2}{3} \cdot \frac{L}{2} \cdot \frac{L}{4} \cdot \frac{L}{2EI} \cdot \frac{9M_o}{8L} - \frac{L}{4} \cdot \frac{L}{2} \cdot \frac{L}{2EI} \cdot \frac{9M_o}{8L} + \frac{L}{4} \cdot \frac{L}{2} \cdot \frac{M_o}{EI} = \frac{M_oL^2}{128EI}$$

$\vec{t}_{C/B} = \frac{M_oL^2}{128EI} \uparrow$ $\quad \vec{y}_c = \overrightarrow{CC'} = -\vec{t}_{C/B}$ $\quad \therefore \boxed{\vec{y}_c = \frac{M_oL^2}{128EI} \downarrow}$