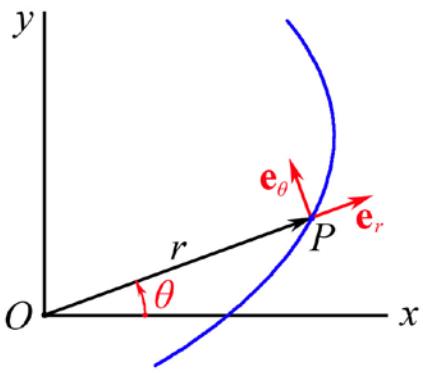
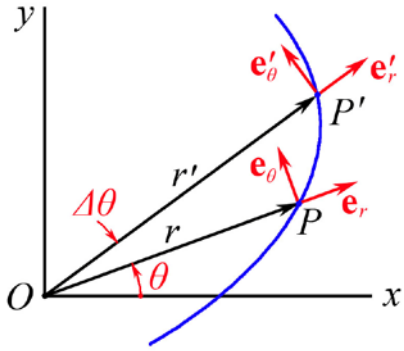
MEEG 4003 Quiz 12.m03.093

1. ⑩ Including sketches, derive the formulas (*a*) $\dot{\mathbf{e}}_r = \dot{\theta}\mathbf{e}_\theta$, (*b*) $\dot{\mathbf{e}}_\theta = -\dot{\theta}\mathbf{e}_r$.

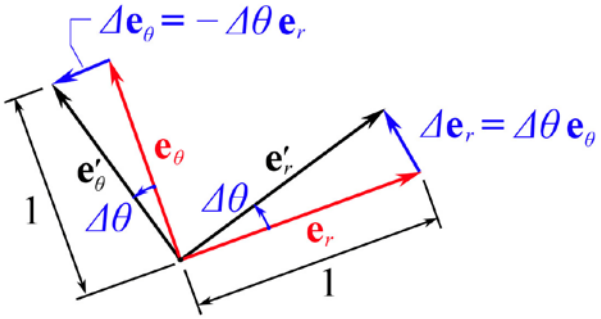
2. ⑩ Starting with the position vector $\mathbf{r} = r\,\mathbf{e}_r$ and using the above formulas, derive the formulas for the velocity **v** and acceleration **a** of a particle in polar coordinates.



1.



②



②

$$\dot{\mathbf{e}}_r = \lim_{\Delta t\to 0}\frac{\Delta \mathbf{e}_r}{\Delta t} = \lim_{\Delta t\to 0}\frac{\Delta\theta}{\Delta t}\mathbf{e}_\theta = \frac{d\theta}{dt}\mathbf{e}_\theta = \dot{\theta}\mathbf{e}_\theta \qquad \text{Q.E.D.} \quad ③$$

$$\dot{\mathbf{e}}_\theta = \lim_{\Delta t\to 0}\frac{\Delta \mathbf{e}_\theta}{\Delta t} = -\lim_{\Delta t\to 0}\frac{\Delta\theta}{\Delta t}\mathbf{e}_r = -\frac{d\theta}{dt}\mathbf{e}_r = -\dot{\theta}\mathbf{e}_r \qquad \text{Q.E.D.} \quad ③$$

2. $\mathbf{r} = r\,\mathbf{e}_r$ $\qquad \mathbf{v} = \dot{\mathbf{r}} = \dot{r}\,\mathbf{e}_r + r\,\dot{\mathbf{e}}_r = \dot{r}\,\mathbf{e}_r + r\,(\dot{\theta}\mathbf{e}_\theta)$ $\qquad \mathbf{v} = \dot{r}\,\mathbf{e}_r + r\dot{\theta}\mathbf{e}_\theta$ ③

$$\mathbf{a} = \dot{\mathbf{v}} = (\ddot{r}\mathbf{e}_r + \dot{r}\dot{\mathbf{e}}_r) + (\dot{r}\dot{\theta}\mathbf{e}_\theta + r\ddot{\theta}\mathbf{e}_\theta + r\dot{\theta}\dot{\mathbf{e}}_\theta) \quad ⑤$$
$$= \ddot{r}\mathbf{e}_r + \dot{r}(\dot{\theta}\mathbf{e}_\theta) + \dot{r}\dot{\theta}\mathbf{e}_\theta + r\ddot{\theta}\mathbf{e}_\theta + r\dot{\theta}(-\dot{\theta}\mathbf{e}_r)$$

$$\mathbf{a} = (\ddot{r} - r\dot{\theta}^2)\mathbf{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{e}_\theta \quad ②$$