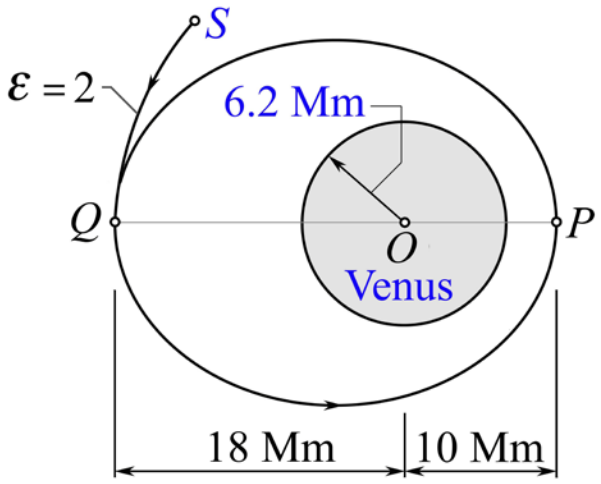
MEEG 4003 Quiz 12.m07.093

⑩ A spacecraft approaches Venus along a hyperbolic trajectory SQ of eccentricity $\varepsilon = 2$ as shown. As it reaches Q, retrorockets are fired momentarily to insert it into an elliptic orbit as indicated. If the mass of Venus is 0.8144 time of the mass of the Earth, determine for the spacecraft (a) its speed $(v_Q)_{\text{hyp}}$ as it approaches Q, (b) its speed $(v_Q)_{\text{ell}}$ after firing of retrorockets at Q, (c) time t_{QP} required to travel from Q to P.



(a) $\dfrac{1}{r_Q} = \dfrac{GM_V}{h_1^2}(1+\varepsilon\cos\theta) = \dfrac{GM_E\,(M_V/M_E)}{r_Q^2\,(v_Q)_{\text{hyp}}^2}(1+\varepsilon\cos\theta), \qquad GM_E = g_E R_E^2$

$$\frac{1}{18\times10^3} = \frac{(9.81\times10^{-3})(6370)^2(0.8144)}{(18\times10^3)^2(v_Q)_{\text{hyp}}^2}(1+2\cos 0)$$

$(v_Q)_{\text{hyp}} = 7.35$ km/s ④

(b) & (c) $\dfrac{1}{r_P} + \dfrac{1}{r_Q} = \dfrac{2GM_V}{h_2^2} = \dfrac{2GM_E\,(M_V/M_E)}{r_Q^2\,(v_Q)_{\text{ell}}^2}$

$$\frac{1}{10\times10^3} + \frac{1}{18\times10^3} = \frac{2(9.81\times10^{-3})(6370)^2(0.8144)}{(18\times10^3)^2(v_Q)_{\text{ell}}^2}$$

$(v_Q)_{\text{ell}} = 3.58668$ km/s

$$t_{QP} = \frac{\tau}{2} = \frac{\pi(r_P+r_Q)\sqrt{r_P r_Q}}{2h_2} = \frac{\pi[(10+18)\times10^3]\sqrt{(10)(18)}\times10^3}{2(18\times10^3)(3.58668)}\ \text{s} = 9140.06\ \text{s}$$

$(v_Q)_{\text{ell}} = 3.59$ km/s ③ $\qquad t_{QP} = 2.54$ hours ③