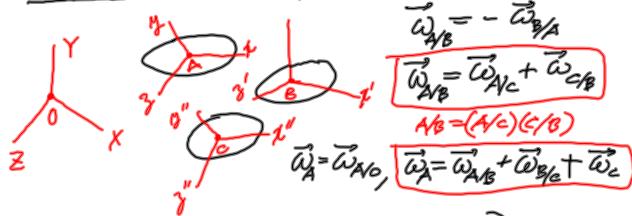
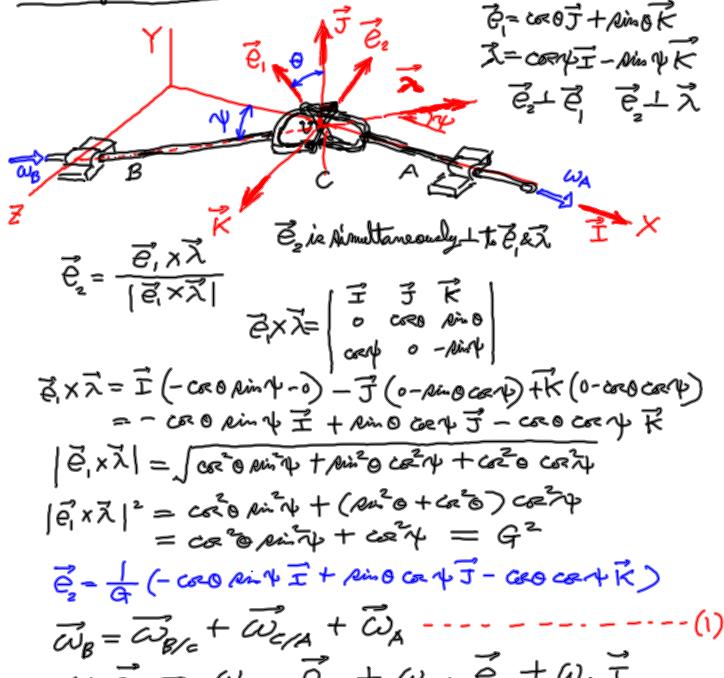


Addition theorem for angular velocities



Example 19.2 (See PP. 842-844.)



$$\begin{aligned} \omega_B (\cos \psi \vec{I} - \sin \psi \vec{K}) \\ = \frac{\omega_B/c}{G} (-\cos \theta \sin \phi \vec{I} + \sin \theta \cos \phi \vec{J} - \cos \phi \vec{K}) \\ + \omega_{C/A} (\cos \theta \vec{J} + \sin \theta \vec{K}) + \omega_A \vec{I} \end{aligned}$$

$$\vec{I}: \omega_B \cos \psi = -\frac{1}{G} \omega_{B/C} \cos \theta \sin \phi + \omega_A \quad \dots (2)$$

$$\vec{J}: 0 = \frac{1}{G} \omega_{B/C} \sin \theta \cos \phi + \omega_{C/A} \cos \theta \quad \dots (3) \times (\sin \theta)$$

$$\vec{K}: -\omega_B \sin \psi = -\frac{1}{G} \omega_{B/C} \cos \theta \cos \phi + \omega_{C/A} \sin \theta \quad \dots (4) \times (-\cos \phi)$$

$$0 = \frac{1}{G} \omega_{B/C} \sin^2 \theta \cos \phi + \omega_{C/A} \cos \theta \sin \phi \quad \dots \dots \dots (3')$$

$$\omega_B \sin \psi \cos \theta = \frac{1}{G} \omega_{B/C} \cos^2 \theta \cos \phi - \omega_{C/A} \sin \theta \cos \theta \quad \dots (4')$$

Adding Eqs. (3') and (4'), side by side, we get

$$\omega_B \sin \psi \cos \theta = \frac{1}{G} \omega_{B/C} \cos \psi, \quad \boxed{\omega_{B/C} = G \omega_B \tan \psi \cos \theta}$$

Substituting $\omega_{B/C}$ into Eq. (2), we write

$$\begin{aligned} \omega_B \cos \psi &= -\frac{1}{G} ((\cancel{\omega_B} \tan \psi \cos \theta) \cos \theta \sin \phi + \omega_A) \\ &= -\omega_B \left(\frac{\sin^2 \psi}{\cos \psi} \cos^2 \theta \right) + \omega_A \end{aligned}$$

$$\omega_B \cos^2 \psi = -\omega_B \sin^2 \psi \cos^2 \theta + \omega_A \cos \psi$$

$$\omega_B (1 - \sin^2 \psi) = \omega_B \sin^2 \psi \cos^2 \theta + \omega_A \cos \psi$$

$$\omega_B - \omega_B \sin^2 \psi = \omega_B \sin^2 \psi \cos^2 \theta + \omega_A \cos \psi$$

$$\omega_B - \omega_B \sin^2 \psi (1 - \cos^2 \theta) = \omega_A \cos \psi$$

$$\omega_B - \omega_B \sin^2 \psi \sin^2 \theta = \omega_A \cos \psi, \quad \omega_B (1 - \sin^2 \psi \sin^2 \theta) = \omega_A \cos \psi$$

$$\therefore \boxed{\frac{\omega_B}{\omega_A} = \frac{\cos \psi}{1 - \sin^2 \psi \sin^2 \theta}}$$

$$\boxed{\frac{\omega_B}{\omega_A} = 1 \text{ when } \psi = 0}$$