Addition theorem for angular velocities


Example 19.2 (see PP. 842-844.)


$$
\begin{aligned}
& \overrightarrow{E_{1}} \times \vec{\lambda}=\vec{I}(-\cos \theta \sin \psi-0)-\vec{J}(0-\sin \theta \cos \psi) \overrightarrow{+K}(0-\cos \theta \cos \psi) \\
&=-\cos \theta \sin \psi \vec{I}+\sin \theta \cos \psi \vec{J}-\cos \theta \cos \psi \vec{K} \\
&\left|\vec{e}_{1} \times \vec{\lambda}\right|=\sqrt{\cos ^{2} \theta \sin ^{2} \psi+\sin ^{2} \theta \cos ^{2} \psi+\cos ^{2} \theta \cos ^{2} \psi} \\
&\left|\vec{e}_{1} \times \vec{\lambda}\right|^{2}=\cos ^{2} \theta \sin ^{2} \psi+\left(\sin ^{2} \theta+\cos ^{2} \theta\right) \cos ^{2} \psi \\
&=\cos ^{2} \theta \sin ^{2} \psi+\cos ^{2} \psi=G^{2} \\
& \vec{e}_{2}=\frac{1}{Q}(-\cos \theta \sin \psi \vec{I}+\sin \theta \cos \psi \vec{J}-\cos \theta \cos \psi \vec{K}) \\
& \vec{\omega}_{B}=\vec{\omega}_{B / C}+\vec{\omega}_{C / A}+\vec{\omega}_{A} \cdots \cdots \\
& \omega_{B} \vec{\lambda}=\omega_{B / C} \vec{e}_{2}+\omega_{c / A} \overrightarrow{\omega_{1}}+\omega_{A} \vec{I}
\end{aligned}
$$

$\omega_{B}(\cos \psi \vec{I}-\sin \psi \vec{K})$
$=\frac{\omega_{B / S}}{G}(-\cos \theta \sin \psi \vec{I}+\sin \theta \cos \psi \vec{J}-\cos \theta \cos \overrightarrow{\psi K})$

$$
+\omega_{c / A}(\cos \theta \vec{J}+\sin \overrightarrow{\theta K})+\omega_{A} \vec{I}
$$

I: $\quad \omega_{B} \cos \psi=-\frac{1}{Q} \omega_{B / C} \cos \theta \sin \psi+\omega_{A}-(2)$

$$
\begin{equation*}
\vec{J}: 0=\frac{1}{ब} \omega_{B / C} \sin \theta \cos \psi+\omega_{C / A} \cos \theta \tag{3}
\end{equation*}
$$

$$
\begin{equation*}
\vec{K}:-\omega_{B} \sin \psi=-\frac{1}{G} \omega_{B / C} \cos \theta \cos \psi+\omega_{c A} \sin \theta \cdots \tag{4}
\end{equation*}
$$

$$
\begin{equation*}
0=\frac{1}{9} \omega_{B / C} \sin ^{2} \theta \cos \psi+\omega_{C / A} \cos \theta \sin \theta \tag{array}
\end{equation*}
$$

$\omega_{B} \sin \psi \cos \theta=\frac{1}{a} \omega_{B / C} \cos ^{2} \theta \cos \psi-\omega_{C / A} \sin \theta \cos \theta \cdots(4)$. $3^{\prime}$ ) and $\left(4^{\prime}\right)$, side by side, we get
adding Egg.

$$
\omega_{B} \sin \psi \cos \theta=\frac{1}{9} \omega_{B / C} C R \psi, \omega_{B / C}=G \omega_{B} \tan \psi \cos \theta
$$

Substituting $\omega_{B / C}$ into eq. (2), we write

$$
\begin{aligned}
& \omega_{B} \cos \psi=-\frac{1}{\operatorname{G}}\left(\frac{1}{A} \omega_{B} \tan \psi \cos \theta\right) \cos \theta \sin \psi+\omega_{A} \\
&=-\omega_{B}\left(\frac{\sin ^{2} \psi}{\cos \psi} \cos ^{2} \theta\right)+\omega_{A} \\
& \omega_{B} \cos ^{2} \psi=-\omega_{B} \sin ^{2} \psi \cos ^{2} \theta+\omega_{A} \cos \psi \\
& \omega_{B}\left(1-\sin ^{2} \psi\right)=\omega_{B} \sin ^{2} \psi \cos ^{2} \theta+\omega_{A} \cos \psi \\
& \omega_{B}-\omega_{B} \sin ^{2} \psi=\omega_{B} \sin ^{2} \psi \cos ^{2} \theta+\omega_{A} \cos \psi \\
& \omega_{B}-\omega_{B} \sin ^{2} \psi\left(1-\cos ^{2} \theta\right)=\omega_{A} \cos \psi \\
& \omega_{B}-\omega_{B} \sin ^{2} \psi \sin ^{2} \theta=\omega_{A} \cos \psi, \quad \omega_{B}\left(1-\sin ^{2} \psi \sin ^{2} \theta\right)=\omega_{A} \cos \psi \\
& \therefore \frac{\omega_{B} \psi}{\omega_{A}}=\frac{\cos ^{2} \psi \sin ^{2} \theta}{\omega_{A}}=1 \omega \operatorname{sen} \psi=0
\end{aligned}
$$

