

$$\vec{v}_B = \vec{v}_{B/Axyz} + \vec{v}_B, \quad \vec{v}_B = \vec{\omega} \times \vec{r}_{BA} + \vec{v}_A$$

$$\vec{a}_B = \vec{a}_{B/Axyz} + \vec{a}_B + 2\vec{\omega} \times \vec{v}_{B/Axyz}$$

$$\vec{a}_B = \vec{\omega} \times \vec{r}_{BA} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{BA}) + \vec{a}_A$$

Addition theorem for angular accelerations

$$\vec{\omega}_{AB} = -\vec{\omega}_{B/A}$$

$$\vec{\omega}_A = \vec{\omega}_{AB} + \vec{\omega}_{B/C} + \vec{\omega}_C$$

$$\vec{\alpha}_{AB} = (\dot{\vec{\omega}}_{AB})_{Bx'y'z'}$$

angular acceleration of A in B

$$\vec{\alpha}_{AC} = (\dot{\vec{\omega}}_{AC})_{Cx''y''z''}$$

angular accel. of A in C

$$\vec{\alpha}_{CB} = (\dot{\vec{\omega}}_{CB})_{Bx'y'z'}$$

" " " C in B

$(\dot{\vec{\omega}}_{AC})_{Bx'y'z'}$ ← one body A & two reference bodies C & B.

Not an angular acceleration of A in C or B.
having no simple physical interpretation

$$\dot{\vec{Q}} = (\dot{\vec{\omega}})_{Axyz} + \vec{\alpha} \times \vec{Q}, \text{ where } \vec{\alpha} = \vec{\omega}_{AO} = (\vec{\omega}_A)_{Oxyz}$$

$$(\dot{\vec{\omega}}_{AC})_{Bx'y'z'} = (\dot{\vec{\omega}}_{AC})_{Cx''y''z''} + \vec{\omega}_{CB} \times \vec{\omega}_{AC}$$

$$= \vec{\alpha}_{AC} + \vec{\omega}_{CB} \times \vec{\omega}_{AC}$$

Add. theorem for $\vec{\omega}$: $\vec{\omega}_{AB} = \vec{\omega}_{AC} + \vec{\omega}_{CB}$

$$(\dot{\vec{\omega}}_{AB})_{Bx'y'z'} = (\dot{\vec{\omega}}_{AC})_{Bx'y'z'} + (\dot{\vec{\omega}}_{CB})_{Bx'y'z'}$$

$$\vec{\alpha}_{AB} = \vec{\alpha}_{AC} + \vec{\omega}_{CB} \times \vec{\omega}_{AC} + \vec{\alpha}_{CB}$$

Addition theorem

$$\vec{\alpha}_{AB} = \vec{\alpha}_{AC} + \vec{\alpha}_{CB} + \vec{\omega}_{CB} \times \vec{\omega}_{AC}$$

for angular acceleration

$$\begin{aligned} \vec{\alpha}_{AO} &= \vec{\alpha}_{AB} + \vec{\alpha}_{BO} + \vec{\omega}_{BO} \times \vec{\omega}_{AB} \\ &= \vec{\alpha}_{AB} + (\vec{\alpha}_{BC} + \vec{\alpha}_{CO} + \vec{\omega}_{CO} \times \vec{\omega}_{BC}) + \vec{\omega}_{BO} \times \vec{\omega}_{AB} \\ &= \vec{\alpha}_{AB} + \vec{\alpha}_{BC} + \vec{\alpha}_{CO} + \vec{\omega}_{BO} \times \vec{\omega}_{AB} + \vec{\omega}_{CO} \times \vec{\omega}_{BC} \end{aligned}$$

$$\vec{\alpha}_A = \vec{\alpha}_{AB} + \vec{\alpha}_{BC} + \vec{\alpha}_C + \vec{\omega}_B \times \vec{\omega}_{AB} + \vec{\omega}_C \times \vec{\omega}_{BC}$$

Example 19.3

$$\vec{v}_B = 10 \vec{i} / \text{ft} \quad \vec{\omega}_R = 3 \vec{i} / \text{rad/s}$$

$$\vec{\alpha}_R = 4 \vec{i} / \text{rad/s}^2 \quad \vec{v}_B = ? \quad \vec{\alpha}_B = ?$$

Let Axyz be embedded in the bent rod R as shown.

$$\vec{\alpha}_A = \vec{\alpha}_R = 3 \vec{i} / \text{rad/s}$$

$$\vec{\alpha}_B = \vec{\alpha}_{B/Axyz} + \vec{\alpha}_B,$$

$$= \frac{10}{5} (4 \vec{i} - 3 \vec{j}) + 4(3) \vec{k}$$

$$\vec{v}_B = 8 \vec{i} - 6 \vec{j} + 12 \vec{k} \text{ ft/s}$$

$$\begin{aligned} \vec{a}_B &= \vec{a}_{B/Axyz} + \vec{a}_B + 2 \vec{\omega} \times \vec{v}_{B/Axyz} \\ &= \frac{(10)^2}{5} \left(\frac{1}{5} (-3 \vec{i} - 4 \vec{j}) + [4(2)^2 (-\vec{j}) + 4(4) \vec{k} + \vec{o}] \right) \\ &\quad + 2(3 \vec{i}) \times \frac{10}{5} (4 \vec{i} - 3 \vec{j}) \\ &= -12 \vec{i} - 52 \vec{j} - 20 \vec{k} \quad \boxed{\vec{a}_B = -12 \vec{i} - 52 \vec{j} - 20 \vec{k} \text{ ft/s}^2} \end{aligned}$$

$$\begin{aligned} \vec{\alpha}_A &= \vec{\alpha}_{AB} + \vec{\alpha}_{BC} + \vec{\alpha}_{CO} + \vec{\alpha}_{EG} + \vec{\alpha}_E \\ &\quad + \vec{\omega}_B \times \vec{\omega}_{AB} + \vec{\omega}_C \times \vec{\omega}_{BC} + \vec{\omega}_D \times \vec{\omega}_{CD} + \vec{\omega}_E \times \vec{\omega}_{DE} \end{aligned}$$

Addition theorem for angular accelerations