MEEG 4103 Quiz 5.1.081

In terms of principal stresses, σ_1, σ_2, σ_3, the *von Mises stress* is

$$\sigma' = \frac{1}{\sqrt{2}}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]^{1/2}$$

It is known that the characteristic equation for the stress matrix using principal axes at a point is

$$-\lambda^3 + (\sigma_1 + \sigma_2 + \sigma_3)\lambda^2 - (\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)\lambda + \sigma_1\sigma_2\sigma_3 = 0$$

and that for the stress matrix using *xyz* axes at the same point is

$$-\lambda^3 + (\sigma_x + \sigma_y + \sigma_z)\lambda^2 - (\sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_z\sigma_x - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{zx}^2)\lambda$$
$$+ (\sigma_x\sigma_y\sigma_z + 2\tau_{xy}\tau_{yz}\tau_{zx} - \sigma_x\tau_{yz}^2 - \sigma_y\tau_{zx}^2 - \sigma_z\tau_{xy}^2) = 0$$

Show that the *von Mises stress* using *xyz* axes is given by

$$\sigma' = \frac{1}{\sqrt{2}}[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)]^{1/2}$$
