MEEG 4103 Quiz 5.1.091 (20 points)

It is known that the total strain energy per unit volume u at a point P of a machine part is given by

$$u = \frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\nu\left(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1\right)\right]$$

Making use of this formula, derive (*a*) the strain energy per unit volume u_v associated with only the volume change at P, (*b*) the distortion strain energy per unit volume u_d at P.

(*a*)

$$\sigma_{\text{av}} = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} \quad ②$$

$$u_v = u\big|_{\sigma_1=\sigma_2=\sigma_3=\sigma_{\text{av}}} = \frac{1}{2E}\left[3\sigma_{\text{av}}^2 - 2\nu(3\sigma_{\text{av}}^2)\right] = \frac{1-2\nu}{2E}(3\sigma_{\text{av}}^2)$$

$$= \frac{1-2\nu}{6E}\left(\sigma_1 + \sigma_2 + \sigma_3\right)^2 \quad ⑦$$

$$u_v = \frac{1-2\nu}{6E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + 2\left(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1\right)\right] \quad ①$$

(*b*)

$$u_d = u - u_v \quad ②$$

$$u_d = \frac{1}{6E}\begin{bmatrix} 3(\sigma_1^2 + \sigma_2^2 + \sigma_3^2) - 6\nu\left(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1\right) \\ -(\sigma_1^2 + \sigma_2^2 + \sigma_3^2) - 2\left(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1\right) \\ +2\nu(\sigma_1^2 + \sigma_2^2 + \sigma_3^2) + 4\nu\left(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1\right) \end{bmatrix}$$

$$= \frac{2(1+\nu)}{6E}\left[(\sigma_1^2 + \sigma_2^2 + \sigma_3^2) - \left(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1\right)\right] \quad ⑦$$

$$u_d = \frac{1+\nu}{3E}\left[\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}{2}\right] \quad ①$$