

MEEG 4103 Quiz 5.1b.081

1. ⑧ It is given that the strain energy per unit volume of a machine part is

$$u = \frac{1}{2E} \left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\nu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1) \right]$$

Show that the distortion strain energy per unit volume of a machine part is

$$u_d = \frac{1+\nu}{6E} [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]$$

2. ② Describe the distortion-energy theory.

1.
$$\sigma_{av} = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3) \quad \text{①}$$

$$u = \frac{1}{2E} \left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\nu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1) \right]$$

$$u_v = u \Big|_{\sigma_1=\sigma_2=\sigma_3=\sigma_{av}} = \frac{1-2\nu}{6E} \left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + 2(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1) \right] \quad \text{③}$$

$$u_d = u - u_v = \frac{1+\nu}{6E} [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2] \quad \text{④}$$

2. The distortion-energy theory predicts that yielding occurs when the distortion strain energy per unit volume reaches or exceeds the distortion strain energy per unit volume for yield in simple tension or compression of the same material. ②