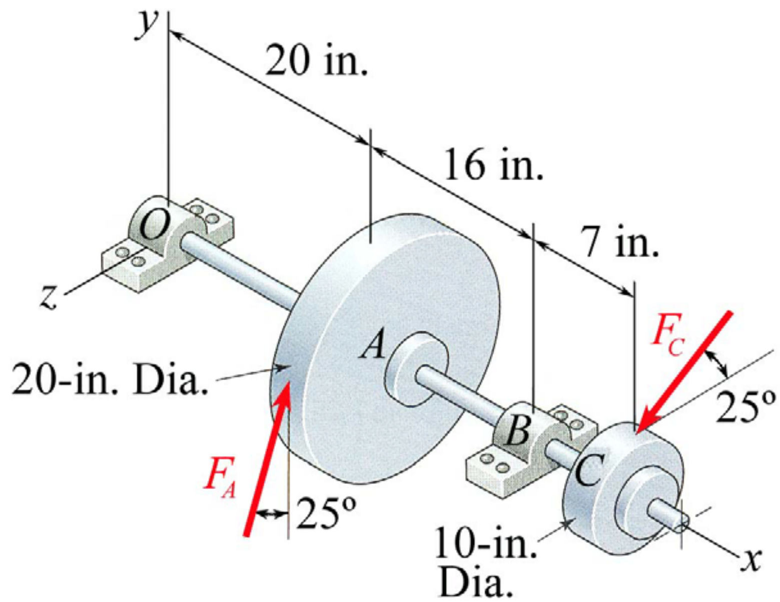
MEEG 4103 Quiz 5.3b.081 (Closed book, closed notes)

1. ④ Derive the octahedral shear stress τ_{oct}.

2. ⑥ The gear forces shown are parallel to the *yz* plane, where F_A= 500 lb, the bearings at *O* and *B* may be taken as simple supports, and the shaft has $S_y = 60$ kpsi. For static analysis and a factor of safety of 3.5, use *distortion-energy theory* to find the minimum safe diameter d of the shaft.



1.

$$\left[\sigma_{ij}\right]=\begin{bmatrix}\sigma_1 & 0 & 0\\ 0 & \sigma_2 & 0\\ 0 & 0 & \sigma_3\end{bmatrix} \text{ (½)} \quad n_i \Rightarrow \frac{1}{\sqrt{3}}\langle 1,\,1,\,1\rangle \text{ (½)} \quad t_i = \sigma_{ji} n_j \Rightarrow \frac{1}{\sqrt{3}}\langle \sigma_1,\ \sigma_2,\ \sigma_3\rangle \text{ ①}$$

$$t^2 = t_i t_i = \frac{1}{3}\left(\sigma_1^2+\sigma_2^2+\sigma_3^2\right) \text{ (½)} \qquad \sigma_{\text{oct}} = t_i n_i = \frac{1}{3}\left(\sigma_1+\sigma_2+\sigma_3\right) \text{ (½)}$$

$$\tau_{\text{oct}}^2 = t^2 - \sigma_{\text{oct}}^2 \text{ (½)} \qquad \therefore\ \tau_{\text{oct}} = \frac{1}{3}\left[\left(\sigma_1-\sigma_2\right)^2+\left(\sigma_2-\sigma_3\right)^2+\left(\sigma_3-\sigma_1\right)^2\right]^{1/2} \text{ (½)}$$

2. $F_A = 500$ lb Draw FBD ① $F_C = 1000$ lb (½) $\mathbf{O} = -283.5775\mathbf{j} + 270.1417\mathbf{k}$ lb ①
(or $\mathbf{B} = 253.042\mathbf{j} - 965.140\mathbf{k}$ lb) $M_A = 7833.077$ lb·in. (½) $M_B = 7000$ lb·in. (½)

<u>**At *A***</u>: $\sigma_x = 9973.38\,r^{-3}$ (½) $\tau_{xy} = -2884.87\,r^{-3}$ (½) $\sigma' = 11155.1\,r^{-3}$ (½) $n = 3.5$
$S_y = 60\times 10^3$ psi $n = S_y/\sigma'$ $r = 0.8666$ in. $d = 1.733$ in. (½) Use $d = 1.75$ in. (½)