UNIVERSITY OF ARKANSAS

MEEG 4703 Mathematical Methods in Engineering

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Texts:

1. ***Index Notation & Cartesian Tensors***, seminar notes, I. C. Jong, 2006
2. ***Advanced Engineering Mathematics**, Fifth Edition*, Zill & Wright, Jones and Bartlett Publishers, 2014

Homework Assignments

Cartesian Tensors

C1. Derive the transformation law for first-order Cartesian tensors: $A_i' = a_{ij} A_j$

C2. Derive the transformation law for first-order Cartesian tensors: $A_i = a_{ji} A_j'$

Using index notation, prove the following identities:

N1. $\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C}) = (\mathbf{A}\times\mathbf{B})\cdot\mathbf{C}$

N2. $\nabla(\mathbf{A}\cdot\mathbf{B}) = (\mathbf{B}\cdot\nabla)\mathbf{A} + (\mathbf{A}\cdot\nabla)\mathbf{B} + \mathbf{B}\times(\nabla\times\mathbf{A}) + \mathbf{A}\times(\nabla\times\mathbf{B})$

N3. $(\mathbf{A}\times\mathbf{B})\cdot(\mathbf{C}\times\mathbf{D}) + (\mathbf{B}\times\mathbf{C})\cdot(\mathbf{A}\times\mathbf{D}) + (\mathbf{C}\times\mathbf{A})\cdot(\mathbf{B}\times\mathbf{D}) = 0$

N4. $(\mathbf{A}\times\mathbf{B})\cdot(\mathbf{B}\times\mathbf{C})\times(\mathbf{C}\times\mathbf{A}) = (\mathbf{A}\cdot\mathbf{B}\times\mathbf{C})^2$

N5. $\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) + \mathbf{B}\times(\mathbf{C}\times\mathbf{A}) + \mathbf{C}\times(\mathbf{A}\times\mathbf{B}) = \mathbf{0}$

N6. $(\mathbf{A}\times\mathbf{B})\cdot(\mathbf{C}\times\mathbf{D}) = (\mathbf{A}\cdot\mathbf{C})(\mathbf{B}\cdot\mathbf{D}) - (\mathbf{A}\cdot\mathbf{D})(\mathbf{B}\cdot\mathbf{C})$

N7. $(\mathbf{A}\times\mathbf{B})\times(\mathbf{C}\times\mathbf{D}) = \mathbf{B}(\mathbf{A}\cdot\mathbf{C}\times\mathbf{D}) - \mathbf{A}(\mathbf{B}\cdot\mathbf{C}\times\mathbf{D}) = \mathbf{C}(\mathbf{A}\cdot\mathbf{B}\times\mathbf{D}) - \mathbf{D}(\mathbf{A}\cdot\mathbf{B}\times\mathbf{C})$

N8. $\nabla\cdot(\nabla\times\mathbf{A}) = 0$

General Tensors

G1. Derive the transformation law for first-order covariant tensors: $A_i = \dfrac{\partial \bar{x}^j}{\partial x^i}\bar{A}_j$

G2. Derive the transformation law for first-order covariant tensors: $\bar{A}_i = \dfrac{\partial x^j}{\partial \bar{x}^i} A_j$

G3. Derive the transformation law for first-order contravariant tensors: $A^i = \dfrac{\partial x^i}{\partial \bar{x}^j}\bar{A}^j$

G4. Derive the transformation law for first-order contravariant tensors: $\bar{A}^i = \dfrac{\partial \bar{x}^i}{\partial x^j} A^j$

Chapter 8 Matrices

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M1. Determine a square root of the matrix **A** as shown.

$$\mathbf{A} = \begin{bmatrix} 5 & -2 \\ -2 & 8 \end{bmatrix}$$

M2. Determine a square root of the matrix **B** as shown.

$$\mathbf{B} = \begin{bmatrix} -12 & 11.5 & -63 \\ -21 & 9.25 & -63 \\ 3 & -3.25 & 18 \end{bmatrix}$$

M3. Determine a square root of the matrix **C** as shown.

$$\mathbf{C} = \begin{bmatrix} -7 & 2.5 & 33 \\ 2.5 & 7.25 & -7.5 \\ -1 & 5 & 7 \end{bmatrix}$$

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Chapter 7 Vectors

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S1. Find the equation of the line which passes through the point (5, 2, 3) and is parallel to the line of intersection of the plane containing the points (1, 1, 1), (1, 2, 4), and (-1, 3, 5) and the plane containing the points (2, 4, 2), (3, -1, 2), and (-2, -4, 3).

Ans. $\frac{x-5}{85} = \frac{y-2}{-33} = \frac{z-3}{-14}$

<u>S2</u>. Find the equation of the plane which contains the points (-1, -1, 1), (1, -1, 2), and (2, 1, -3).
Ans. $2x - 11y - 4z - 5 = 0$

<u>S3</u>. Find the shortest distance from the point (2, 4, 5) to the plane which contains the points (-1, -1, 1), (1, -1, 2), and (2, 1, -3).
Ans. $65/\sqrt{141}$ or 5.47

<u>S4</u>. Find the shortest distance between the line passing through the points $P(5, 0, 0)$ and $Q(4, 8, -4)$ and the line passing through the points $R(0, 0, 12)$ and $S(4, 7, 8)$.
Ans. $448/\sqrt{1937}$ or 10.18

<u>S5</u>. Solve Prob. <u>S-4</u> for $P(5, 0, 0)$, $Q(3, 2, 1)$, $R(0, 0, 4)$ and $S(2, 6, 1)$.
Ans. $1/\sqrt{26}$ or 0.1961

<u>S6</u>. Prove that the line joining the midpoints of two sides of a triangle is parallel to the third side and has one half of its magnitude.

<u>S7</u>. Prove that the medians of a triangle trisect themselves.

<u>S8</u>. Find the shortest distance from the point (6, -4, 4) to the line passing through the points (2, 1, 2) and (3, -1, 4).
Ans. 3

<u>S9</u>. A force $\mathbf{F} = 6x\,\mathbf{j}$ N acts on a particle during its motion from $A_1(0, 0)$ to $A_2(1, 1)$ m along the path C defined by $y = x$. Determine the work done by $\mathbf{F}$ on the particle.
Ans. 3 J

<u>S10</u>. Solve Prob. <u>S-9</u> if the path C is defined by $y = x(2 - x)$.
Ans. 2 J

Chapter 9 Vector Calculus

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