UNIVERSITY of ARKANSAS
1871 ®

MEEG 4703 **Name:** _____

Final Exam **ID#:** _____

1. (20%) Using matrix diagonalization, *identify* and *graph* the conic section

$$2x^2 - 4xy - y^2 = 6$$

2. (20%) Verify Green's theorem

$$\oint_C P\,dx + Q\,dy = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA$$

where $P = x + y^2$, $Q = 2x^2y$, and C is the triangle with vertices (0, 0), (2, 0), and (2, 3).

3. (30%) Verify Stokes' theorem

$$\oint_C \mathbf{F} \cdot d\mathbf{r} = \iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, dS$$

where $\mathbf{F} = x^2y\,\mathbf{i} + y^2\,\mathbf{j} + xyz\,\mathbf{k}$ and S is the part of the cylindrical surface $z = 4 - y^2$ in the first octant and bounded by the planes $y = 2$ and $2x - 3y = 0$.

4. (30%) Verify the divergence theorem

$$\iint_S \mathbf{F} \cdot \mathbf{n}\, dS = \iiint_D \nabla \cdot \mathbf{F}\, dV$$

where $\mathbf{F} = x^2y\,\mathbf{i} + y^2\,\mathbf{j} + xyz\,\mathbf{k}$ and D is the region in the first octant bounded by the surface $z = 4 - y^2$, the xy and yz coordinate planes, as well as the planes $y = 2$ and $2x - 3y = 0$.