MEEG 4703 Quiz m2.183

1. ⑩ In an experiment, the following correspondence was found between temperature T (in °C) and electrical resistance R (in MΩ):

T	400	450	500	550	600	650
R	0.47	0.90	2.0	3.7	7.5	15

Find the least squares line $R = aT + b$. Use this line to estimate the resistance at $T = 700$.

2. ⑩ Identify and graph the given conic section

$$5x^2 - 2xy + 5y^2 = 24$$

1.

We have $\mathbf{Y}^T = (0.47 \quad 0.90 \quad 2.0 \quad 3.7 \quad 7.5 \quad 15)$ and $\mathbf{A}^T = \begin{pmatrix} 400 & 450 & 500 & 550 & 600 & 650 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$.

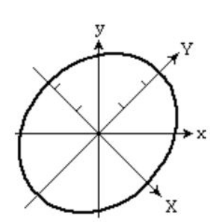
Now $\mathbf{A}^T\mathbf{A} = \begin{pmatrix} 1697500 & 3150 \\ 3150 & 6 \end{pmatrix}$ and $(\mathbf{A}^T\mathbf{A})^{-1} = \dfrac{1}{262500}\begin{pmatrix} 6 & -3150 \\ -3150 & 1697500 \end{pmatrix}$

so $\mathbf{X} = (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{Y} = \begin{pmatrix} 0.0538 \\ -23.3167 \end{pmatrix}$ and the least squares line is $R = 0.0538T - 23.3167$. At $T = 700$, $R \approx 14.3433$.

2.

The given equation can be written as $\mathbf{X}^T\mathbf{A}\mathbf{X} = 24$: $(x \quad y)\begin{pmatrix} 5 & -1 \\ -1 & 5 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 24$. Using $\lambda_1 = 6$, $\lambda_2 = 4$, $\mathbf{K}_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$, $\mathbf{K}_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $\mathbf{P} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$ and $\mathbf{X} = \mathbf{P}\mathbf{X}'$ we find

$$(X \quad Y)\begin{pmatrix} 6 & 0 \\ 0 & 4 \end{pmatrix}\begin{pmatrix} X \\ Y \end{pmatrix} = 24 \quad \text{or} \quad 6X^2 + 4Y^2 = 24.$$



The conic section is an ellipse. Now from $\mathbf{X}' = \mathbf{P}^T\mathbf{X}$ we see that the XY-coordinates of $(1, -1)$ and $(1, 1)$ are $(\sqrt{2}, 0)$ and $(0, \sqrt{2})$, respectively. From this we conclude that the X-axis and Y-axis are as shown in the accompanying figure.