

MEEG 4703 Quiz m4.073

1. (10 points) Determine a square root of the matrix \mathbf{A} as shown.

$$\mathbf{A} = \begin{bmatrix} 5 & -2 \\ -2 & 8 \end{bmatrix}$$

2. (10 points) Using orthogonal matrix and diagonalization, **identify** and **graph** (to scale) the conic section

$$-3x^2 + 8xy + 3y^2 = 25$$

1. $\left(\sqrt{\mathbf{A}} = \mathbf{M} \sqrt{\mathbf{S}} \mathbf{M}^{-1}, \lambda_1 = 9, \lambda_2 = 4 \right)$

$$\mathbf{M} = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}, \mathbf{M}^{-1} = \frac{1}{5} \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$$

$$\sqrt{\mathbf{A}} = \begin{bmatrix} 2.2 & -0.4 \\ -0.4 & 2.8 \end{bmatrix}$$

2. $\lambda_1 = 5, \lambda_2 = -5$

Hyperbola: $X^2 - Y^2 = 5$

$$\mathbf{P} = \frac{1}{\sqrt{5}} \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$$

$$\mathbf{R} = \mathbf{P}^T = \frac{1}{\sqrt{5}} \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$$

