MEEG 4703 Quiz T4.073

1. (10 pts) Derive the following transformation law for first-order covariant tensors:

$$\bar{A}_i = \frac{\partial x^j}{\partial \bar{x}^i} A_j$$

2. (10 pts) Let $\mathbf{A} = z\,\mathbf{i} + 3x\,\mathbf{j} - 2y\,\mathbf{k}$ be a vector in a Cartesian coordinate system $Oxyz$. Using the *laws of transformation for first-order tensors*, determine the covariant component $\bar{A}_2$ and contravariant component $\bar{A}^2$ of this vector in the cylindrical coordinate system $O\rho\phi z$.

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2. $x^1 = x = \rho\cos\phi \qquad x^2 = y = \rho\sin\phi \qquad x^3 = z$

$$\bar{x}^1 = \rho = (x^2 + y^2)^{1/2} \qquad \bar{x}^2 = \phi = \tan^{-1}(yx^{-1}) \qquad \bar{x}^3 = z$$

$$A_1 = A^1 = z \qquad A_2 = A^2 = 3\rho\cos\phi \qquad A_3 = A^3 = -2\rho\sin\phi$$

$$\bar{A}_2 = \frac{\partial x^j}{\partial \bar{x}^2} A_j = \frac{\partial x^1}{\partial \bar{x}^2} A_1 + \frac{\partial x^2}{\partial \bar{x}^2} A_2 + \frac{\partial x^3}{\partial \bar{x}^2} A_3 = \frac{\partial x}{\partial \phi} A_1 + \frac{\partial y}{\partial \phi} A_2 + \frac{\partial z}{\partial \phi} A_3$$

$$= (-\rho\sin\phi)(z) + (\rho\cos\phi)(3\rho\cos\phi) + 0$$

$$\bar{A}_2 = -\rho z\sin\phi + 3\rho^2\cos^2\phi$$

$$\bar{A}^2 = \frac{\partial \bar{x}^2}{\partial x^j} A^j = \frac{\partial \bar{x}^2}{\partial x^1} A^1 + \frac{\partial \bar{x}^2}{\partial x^2} A^2 + \frac{\partial \bar{x}^2}{\partial x^3} A^3$$

$$= \frac{\partial \phi}{\partial x}(z) + \frac{\partial \phi}{\partial y}(3\rho\cos\phi) + \frac{\partial \phi}{\partial z}(-2\rho\sin\phi)$$

$$= \frac{-x^{-2}y}{1 + y^2x^{-2}}(z) + \frac{x^{-1}}{1 + y^2x^{-2}}(3\rho\cos\phi) + 0$$

$$= \frac{-yz}{x^2 + y^2} + \frac{x}{x^2 + y^2}(3\rho\cos\phi) + 0 = -\frac{(\rho\sin\phi)z}{\rho^2} + \frac{3\rho^2\cos^2\phi}{\rho^2}$$

$$\bar{A}^2 = -\frac{z}{\rho}\sin\phi + 3\cos^2\phi$$