

MEEG 4703 Quiz v2.073

(20 points) Verify Stokes' theorem

$$\oint_C \mathbf{F} \cdot d\mathbf{r} = \iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n} dS$$

where $\mathbf{F} = xy\mathbf{i} + yz\mathbf{j} + xz\mathbf{k}$ and S is the part of the cylinder $z = 1 - x^2$ for $0 \leq x \leq 1$, $-2 \leq y \leq 2$.

$$\begin{aligned} & \oint_C xydx + yzdy + xzdz \\ &= \iint_S (-y\mathbf{i} - z\mathbf{j} - x\mathbf{k}) \cdot \frac{2x\mathbf{i} + \mathbf{k}}{\sqrt{4x^2 + 1}} \cdot \sqrt{4x^2 + 1} dy dx \end{aligned}$$

$$0 - \frac{11}{15} + 0 - \frac{19}{15} = -2$$