UNIVERSITY of ARKANSAS

1871

MEEG 4703 **Name:** ____________________

(Underline your **last name**.)

Test I **ID#:** ____________________

1. (20%) Making use of the *laws of transformation* for Cartesian tensors, prove the orthonormal condition that

$$a_{ki}\,a_{kj} = \delta_{ij}$$

2. (20%) Using index notation, prove the identity

$$(\mathbf{D}\times\mathbf{E})\cdot(\mathbf{E}\times\mathbf{F})\times(\mathbf{F}\times\mathbf{D}) = (\mathbf{D}\cdot\mathbf{E}\times\mathbf{F})^2$$

3. (20%) Using index notation, prove the identity

$$(\mathbf{A}\times\mathbf{B})\times(\mathbf{C}\times\mathbf{D}) = \mathbf{B}(\mathbf{A}\cdot\mathbf{C}\times\mathbf{D}) - \mathbf{A}(\mathbf{B}\cdot\mathbf{C}\times\mathbf{D}) = \mathbf{C}(\mathbf{A}\cdot\mathbf{B}\times\mathbf{D}) - \mathbf{D}(\mathbf{A}\cdot\mathbf{B}\times\mathbf{C})$$

4. (20%) The rotation from the $Oy_1y_2y_3$ Cartesian coordinate system to the $Oy_1'y_2'y_3'$ Cartesian coordinate system is defined by the rotation matrix

$$\left[a_{ij}\right] = \begin{bmatrix} 1/3 & -2/3 & 2/3 \\ 2/3 & 2/3 & 1/3 \\ -2/3 & 1/3 & 2/3 \end{bmatrix}$$

If $A_i' \Rightarrow (6, 9, -3)$ and $B_i \Rightarrow (3, 9, -6)$, determine A_i and B_i'.

5. (20%) Derive the transformation laws for first-order tensors:

$$(a)\ \ \bar{A}^i = \frac{\partial \bar{x}^i}{\partial x^j} A^j \qquad\qquad (b)\ \ \bar{A}_i = \frac{\partial x^j}{\partial \bar{x}^i} A_j$$