UNIVERSITY of ARKANSAS

MEEG 4703 **Name:** ______________________

(Underline your **last name**.)

Test I **ID#:** ______________________

1. (20%) Using index notation, prove the identity

$$(\mathbf{P}\times\mathbf{Q})\cdot(\mathbf{Q}\times\mathbf{R})\times(\mathbf{R}\times\mathbf{P})=(\mathbf{P}\cdot\mathbf{Q}\times\mathbf{R})^2$$

2. (20%) Using index notation, prove the identity

$$(\mathbf{A}\times\mathbf{B})\times(\mathbf{C}\times\mathbf{D})=\mathbf{B}(\mathbf{A}\cdot\mathbf{C}\times\mathbf{D})-\mathbf{A}(\mathbf{B}\cdot\mathbf{C}\times\mathbf{D})=\mathbf{C}(\mathbf{A}\cdot\mathbf{B}\times\mathbf{D})-\mathbf{D}(\mathbf{A}\cdot\mathbf{B}\times\mathbf{C})$$

3. (20%) The rotation from the $Oy_1y_2y_3$ Cartesian coordinate system to the $Oy_1'y_2'y_3'$ Cartesian coordinate system is defined by the rotation matrix

$$\left[a_{ij}\right]=\begin{bmatrix} 1/3 & -2/3 & 2/3 \\ 2/3 & 2/3 & 1/3 \\ -2/3 & 1/3 & 2/3 \end{bmatrix}$$

If $A_i \Rightarrow (6, 9, -3)$ and $B_i' \Rightarrow (3, 9, -6)$, determine A_i' and B_i.

4. (20%) Derive the transformation laws for first-order tensors:

$$(a)\ \bar{A}^i=\frac{\partial \bar{x}^i}{\partial x^j}A^j \qquad\qquad (b)\ \bar{A}_i=\frac{\partial x^j}{\partial \bar{x}^i}A_j$$

5. (20%) Let $\mathbf{A}=z\,\mathbf{i}+3x\,\mathbf{j}-5y\,\mathbf{k}$ be a vector in a Cartesian coordinate system $Oxyz$. Using the *laws of transformation for first-order tensors*, determine for this vector in the cylindrical coordinate system $O\rho\phi z$ (a) the covariant components $\bar{A}_1$, $\bar{A}_2$, and $\bar{A}_3$; (b) the contravariant components $\bar{A}^1$, $\bar{A}^2$, and $\bar{A}^3$.