



MEEG 4703

Name: _____
(Underline your last name.)

Test II

ID#: _____

1. (20%) A third-order determinant D is shown. Making use of properties of a determinant, (a) transform the determinant into a third-order determinant whose elements below the main diagonal are zero, (b) show that $D = (b - a)(c - a)(c - b)$.

$$D = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$$

2. (20%) Using orthogonal matrix and diagonalization, **identify** and **graph** (to scale) the conic section

$$125x^2 + 600xy + 720y^2 - 156x + 65y = 338$$

3. (20%) In an experiment, the following correspondence was found between temperature T (in °C) and the coefficient of static friction μ_s :

T	20	40	60	80	100
μ_s	0.22	0.20	0.18	0.17	0.15

Find the least square line $\mu_s = aT + b$. Use this line to estimate the value of μ_s for $T = 120^\circ\text{C}$.

4. (20%) Using Cayley-Hamilton theorem, compute \mathbf{A}^{11} for the matrix \mathbf{A} shown.

$$\mathbf{A} = \begin{bmatrix} -2 & 4 \\ -1 & 3 \end{bmatrix}$$

5. (20%) The eigenvalues of the matrix \mathbf{A} shown are $\lambda_1 > \lambda_2 > \lambda_3 = 2.25$. Find (a) the values of λ_1 and λ_2 , (b) the modal matrix \mathbf{M} , (c) the inverse \mathbf{M}^{-1} , (d) a square root $\sqrt{\mathbf{A}}$.

$$\mathbf{A} = \begin{bmatrix} -12 & 11.5 & -63 \\ -21 & 9.25 & -63 \\ 3 & -3.25 & 18 \end{bmatrix}$$