UNIVERSITY of ARKANSAS
1871

MEEG 4703 **Name:** _____

Test III **ID#:** _____

1. (30%) Verify Green's theorem

$$\oint_C P\,dx + Q\,dy = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA$$

where $P = x$, $Q = x^2 y$, and C is the boundary of the triangle with vertices at (2, 0), (2, 1), and (0, 1).

2. (30%) Verify Stokes' theorem

$$\oint_C \mathbf{F} \cdot d\mathbf{r} = \iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, dS$$

where $\mathbf{F} = yz\,\mathbf{j} + x\mathbf{k}$ and S is the part of the cylindrical surface $z = 4 - y^2$ in the first octant and bounded by the planes $y = 2$ and $2x - 3y = 0$.

3. (40%) Verify the divergence theorem

$$\iint_S \mathbf{F} \cdot \mathbf{n}\, dS = \iiint_D \nabla \cdot \mathbf{F}\, dV$$

where $\mathbf{F} = yz\,\mathbf{j} + x\mathbf{k}$ and D is the region in the first octant bounded by the surface $z = 4 - y^2$, the xy and yz coordinate planes, as well as the planes $y = 2$ and $2x - 3y = 0$.