UNIVERSITY of ARKANSAS
1871

MEEG 4703 **Name:** _____

Test III **ID#:** _____

1. (20%) Use Green's theorem

$$\oint_C P\,dx + Q\,dy = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA$$

to evaluate the integral

$$I = \iint_R x^2 dA$$

by means of a line integral if R is the region bounded by the ellipse

$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

2. (30%) Verify Stokes' theorem

$$\oint_C \mathbf{F} \cdot d\mathbf{r} = \iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, dS$$

where $\mathbf{F} = x^2 y\,\mathbf{i} + (x + y^2)\,\mathbf{j} + xy^2 z\,\mathbf{k}$ and S is the part of the cylindrical surface $z = 4 - y^2$ in the first octant and bounded by the planes $y = 2$ and $2x - 3y = 0$.

3. (30%) Verify the divergence theorem

$$\iint_S \mathbf{F} \cdot \mathbf{n}\, dS = \iiint_D \nabla \cdot \mathbf{F}\, dV$$

where $\mathbf{F} = yz\,\mathbf{j} + x\,\mathbf{k}$ and D is the region in the first octant bounded by the surface $x + y + z = 1$ and the coordinate planes.

4. (20%) Submit your solutions for the following homework problems:

(*a*) Problem 10, on page 538.

(*b*) Problem 6, on page 555.