Teaching Students Work and Virtual Work Method in Statics: A Guiding Strategy with Illustrative Examples

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Mechanics Division

Work $\neq$ Energy? What is work?

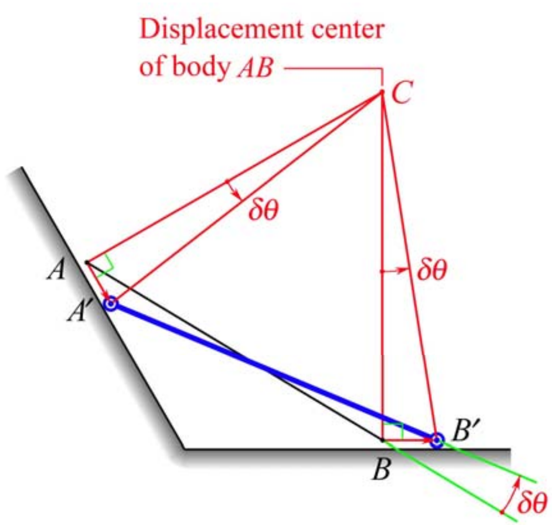
Work is energy in transition to a system due to *force* or *moment* acting on the system through a displacement.

(Note: **Heat** is energy in transition to a system due to *temperature difference* between the system and its surroundings.)

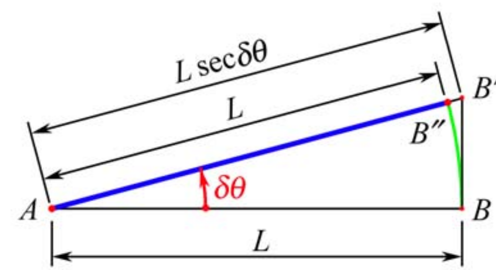
Work differs from energy in that **work** is *not* a property possessed by a system, while **energy** (e.g., kinetic energy or potential energy) is. *Work is a boundary phenomenon.*

$$U_{1\to 2} = \mathbf{F}\cdot\mathbf{q} = Fq_{\|} \qquad\qquad U_{1\to 2} = M(\Delta\theta)$$

Displacement center, rigid-body virtual displace-ment, & compatible virtual displacement

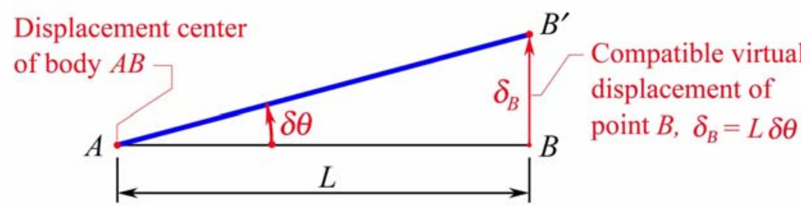


A body undergoes virtual displacement from position *AB* to position *A′B′*. The **displacement center** is at ***C***.



$$\overline{B''B'} \approx \frac{L}{2}(\delta\theta)^2$$

A body undergoes **rigid-body virtual displacement** from *AB* to *AB″*. The *displacement center* is at ***A***.



A body undergoes **compatible virtual displacement** from *AB* to *AB′*. The *displacement center* is at ***A***.

Virtual work

Virtual work is the work done by a force or moment on a body during a *virtual displacement* of the body.

Principle of virtual work

If a body is in equilibrium, the total virtual work δU of the external force system acting on its free body during any *compatible virtual displacement* of its free body is equal to zero; i.e.,

$$\delta U = 0$$

Note that the body in this principle may be a particle, a set of connected particles, a rigid body, or a system of pin-connected rigid bodies (e.g., a frame or machine).

Method of virtual work

Three major steps:

Step 1: Draw the free-body (*FBD*) diagram.

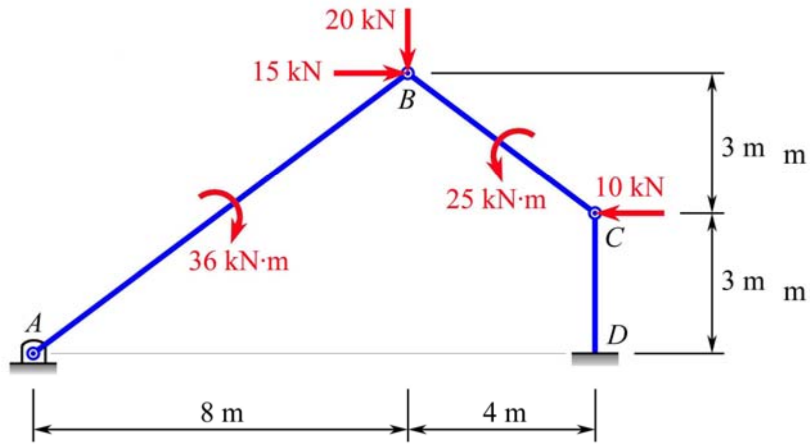
Step 2: Draw the virtual-displacement diagram (*VDD*) with a guiding strategy.

Step 3: Set to zero the total virtual work done to solve for the unknown.

One guiding strategy:

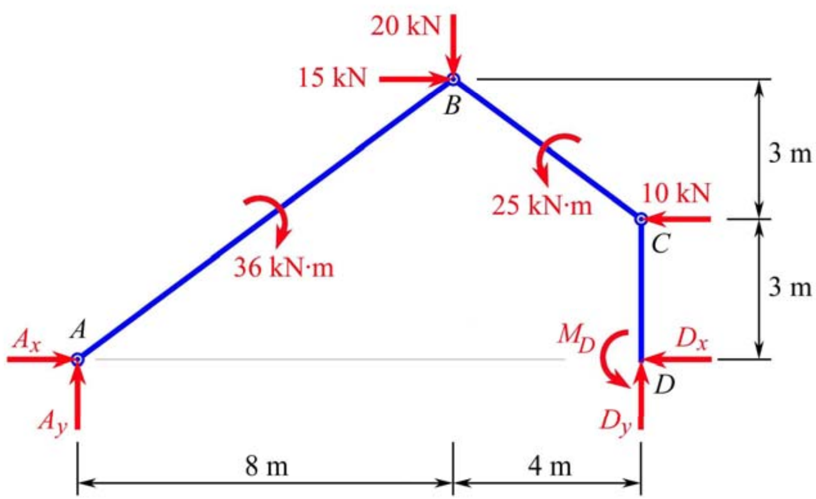
In step 2, give the free body a *compatible virtual displacement* in such a way that the *one* specified unknown, but *no other unknowns*, will be involved in the virtual work done.

Example 1. Determine the horizontal reaction force $\mathbf{D}_x$ at the fixed support *D* of the frame loaded as shown.

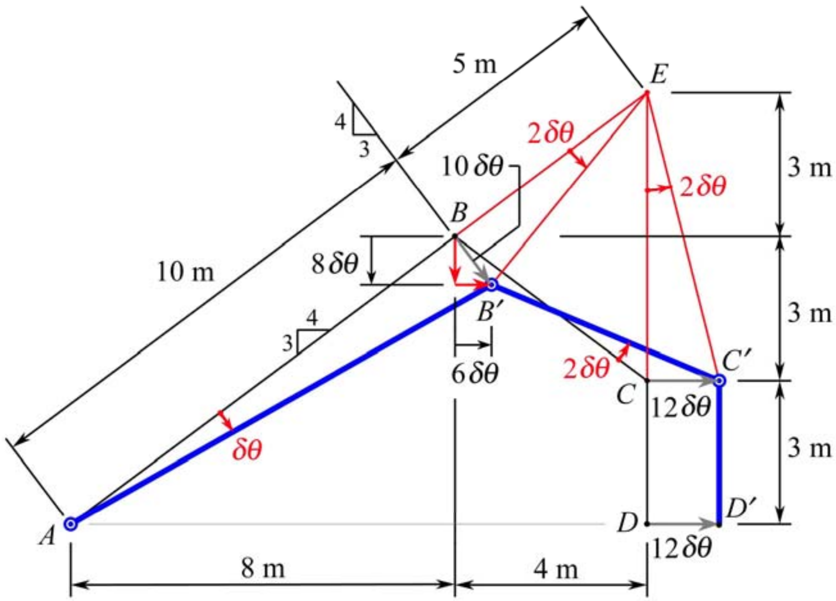


Solution.

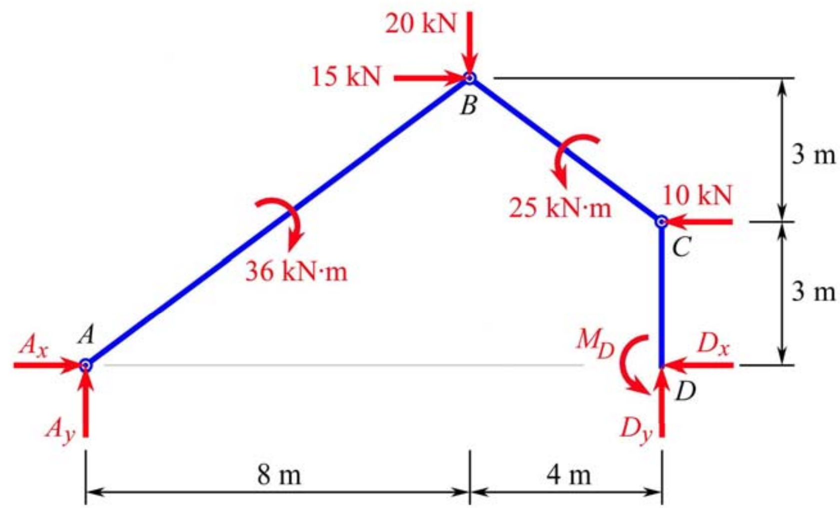
Step 1: ***FBD***



Step 2: *VDD* to involve D_x in δU



(*FBD* repeated here for reference only)



Step 3: $\delta U = 0$:

$$36(\delta\theta) + 15(6\,\delta\theta) + 20(8\,\delta\theta) + 25(2\,\delta\theta) + 10(-12\,\delta\theta) + D_x(-12\,\delta\theta) = 0$$

$$D_x = 18$$

$$\mathbf{D}_x = 18\text{ kN } \leftarrow$$

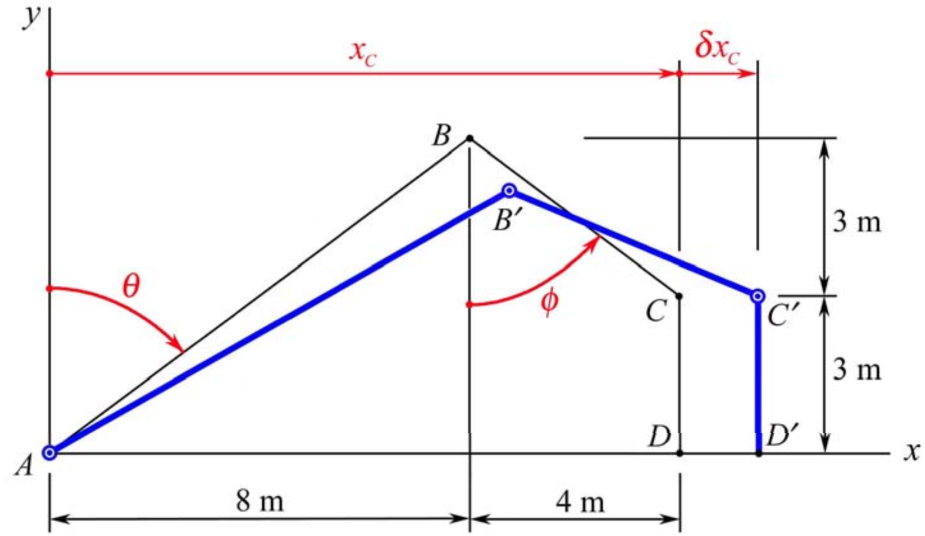
If "*displacement center*" is *not* used to find δx_C, then …

$$\overline{AB}\cos\theta - \overline{BC}\cos\phi = \overline{CD}$$

$$-\overline{AB}(\sin\theta)\,\delta\theta + \overline{BC}(\sin\phi)\,\delta\phi = 0$$

$$\delta\phi = \frac{\overline{AB}\sin\theta}{\overline{BC}\sin\phi}\,\delta\theta$$

$$\delta\phi = \frac{10(4/5)}{5(4/5)}\,\delta\theta = 2\,\delta\theta$$

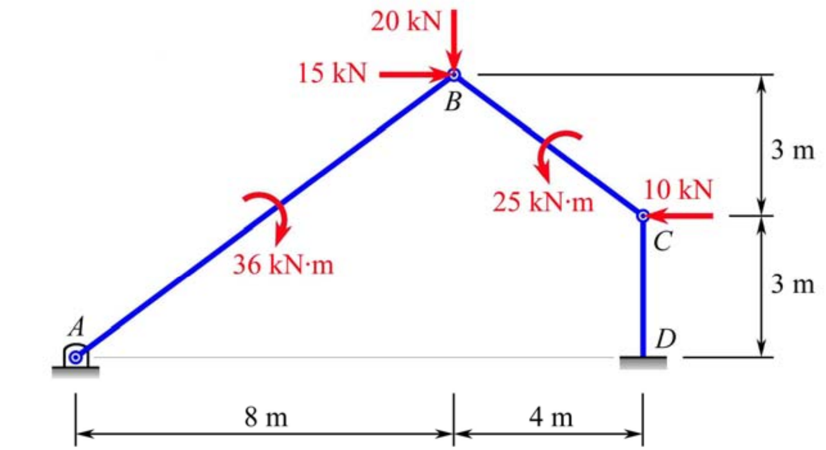


$$x_C = \overline{AB}\sin\theta + \overline{BC}\sin\phi = 10\sin\theta + 5\sin\phi$$

$$\delta x_C = 10(\cos\theta)\,\delta\theta + 5(\cos\phi)\,\delta\phi = 10(3/5)\,\delta\theta + 5(3/5)(2\,\delta\theta) = 12\,\delta\theta$$

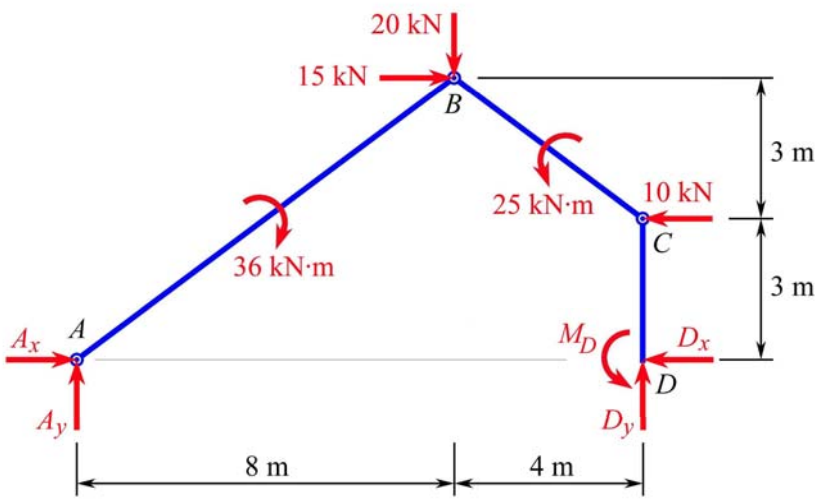
$$\therefore \quad \delta x_C = 12\,\delta\theta$$

Example 2. Determine the reaction moment $\mathbf{M}_D$ at the fixed support D of the frame loaded as shown.

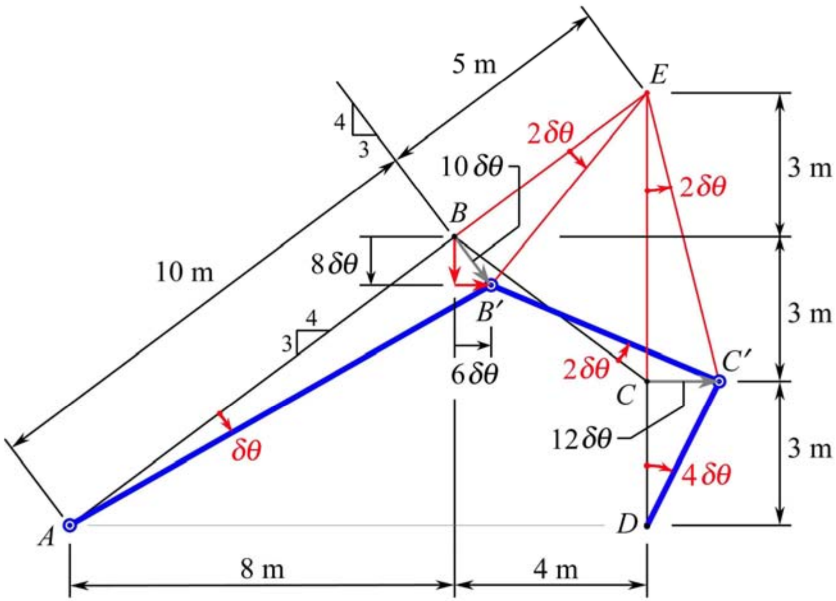


Solution.

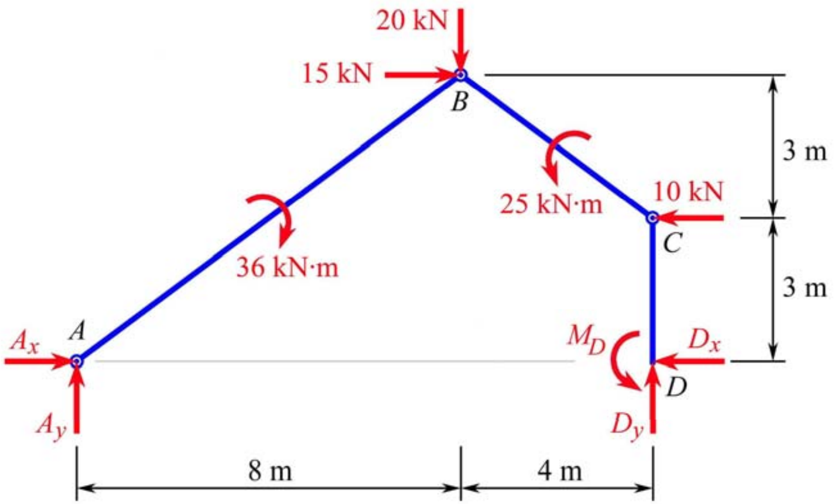
Step 1: ***FBD***



Step 2: *VDD* to involve M_D in δU



(*FBD* repeated here for reference only)



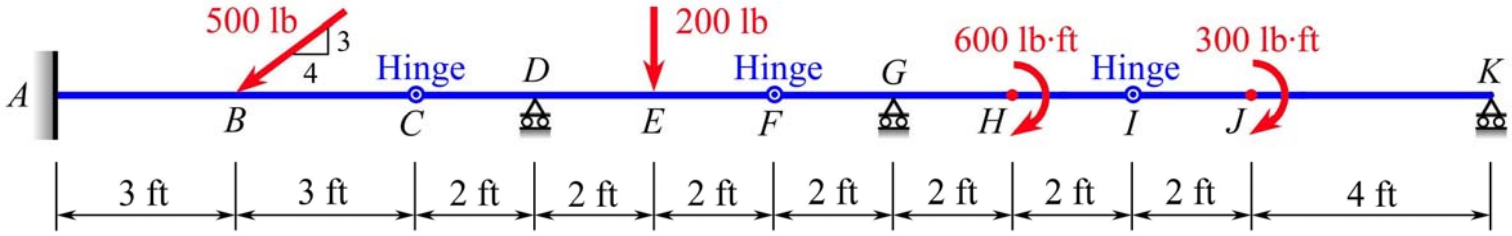
Step 3: $\delta U = 0$:

$$36(\delta\theta) + 15(6\,\delta\theta) + 20(8\,\delta\theta) + 25(2\,\delta\theta) + 10(-12\,\delta\theta) + M_D(-4\,\delta\theta) = 0$$

$$M_D = 54$$

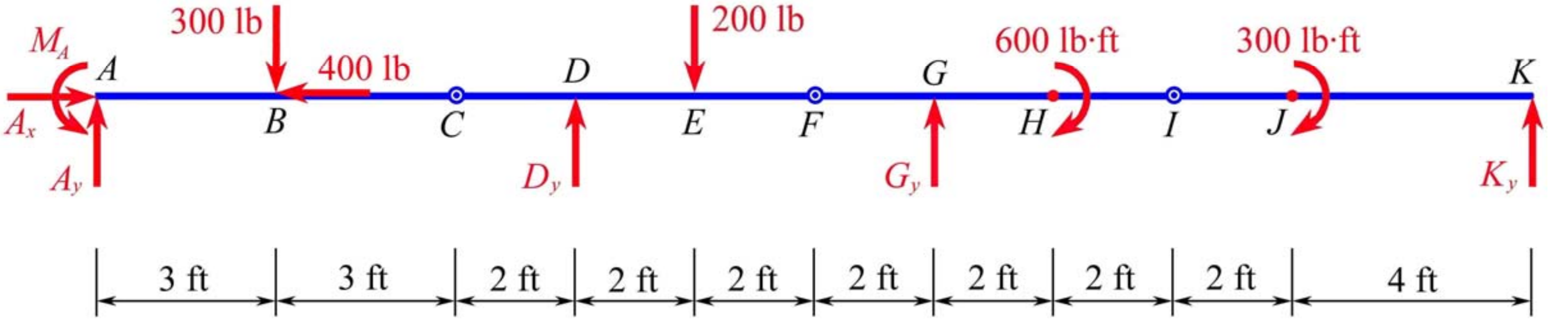
$$\mathbf{M}_D = 54 \text{ kN·m } \circlearrowleft$$

Example 3. Determine the reaction moment $\mathbf{M}_A$ at the fixed support A of the combined beam as shown.

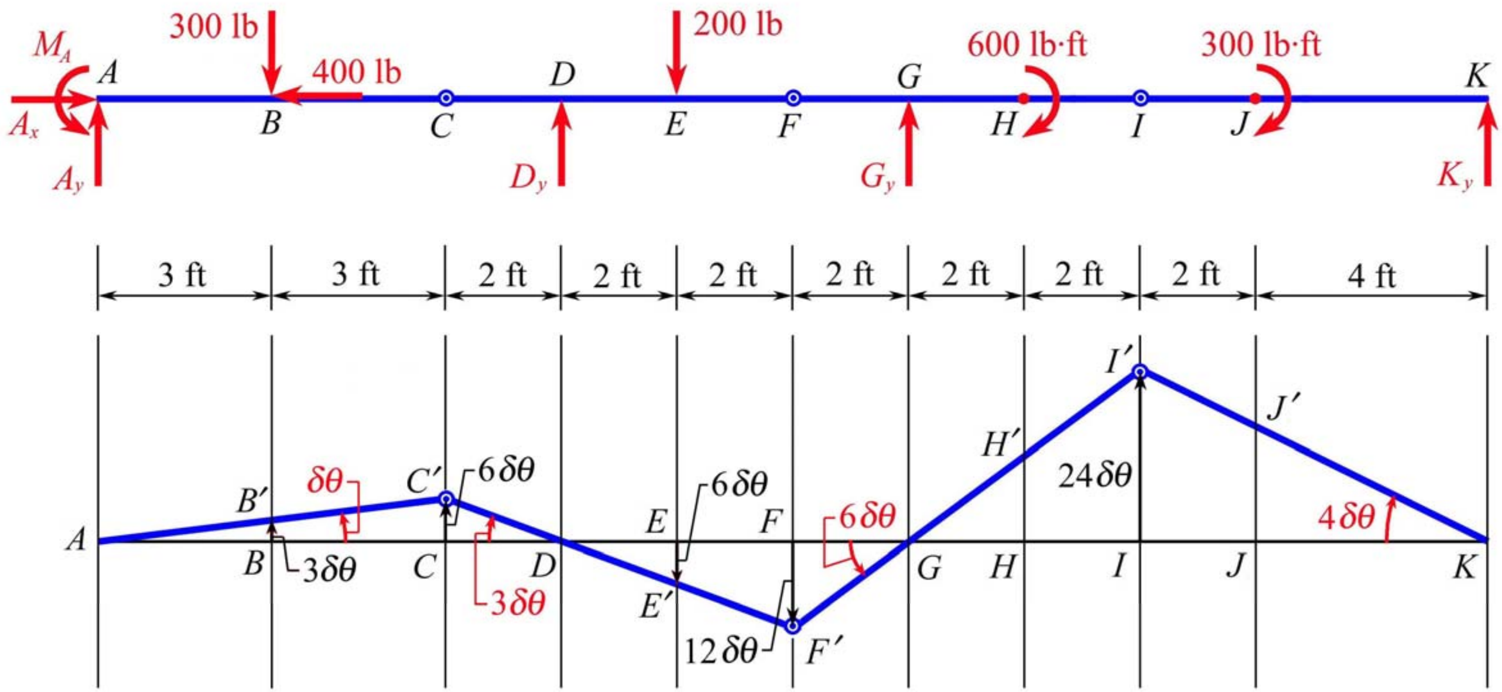


Solution.

Step 1: *FBD*



Step 2: *VDD* to involve M_A in δU (*FBD* repeated here for reference only)

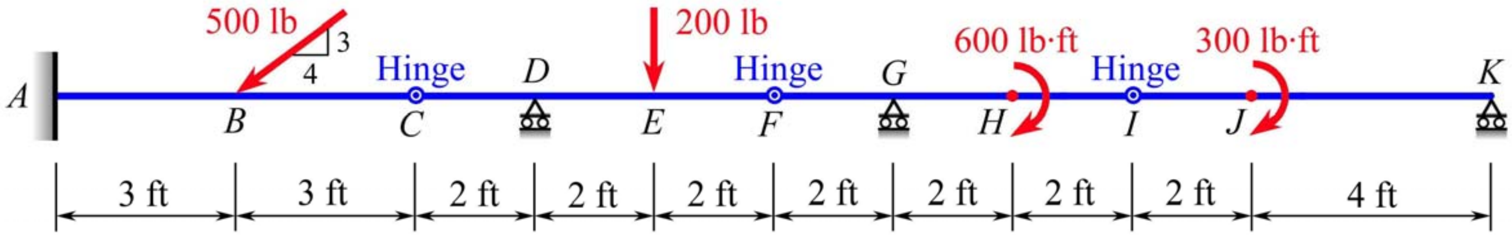


Step 3: $\delta U = 0$:

$$M_A(\delta\theta) + 300(-3\delta\theta) + 200(6\delta\theta) + 600(-6\delta\theta) + 300(4\delta\theta) = 0$$

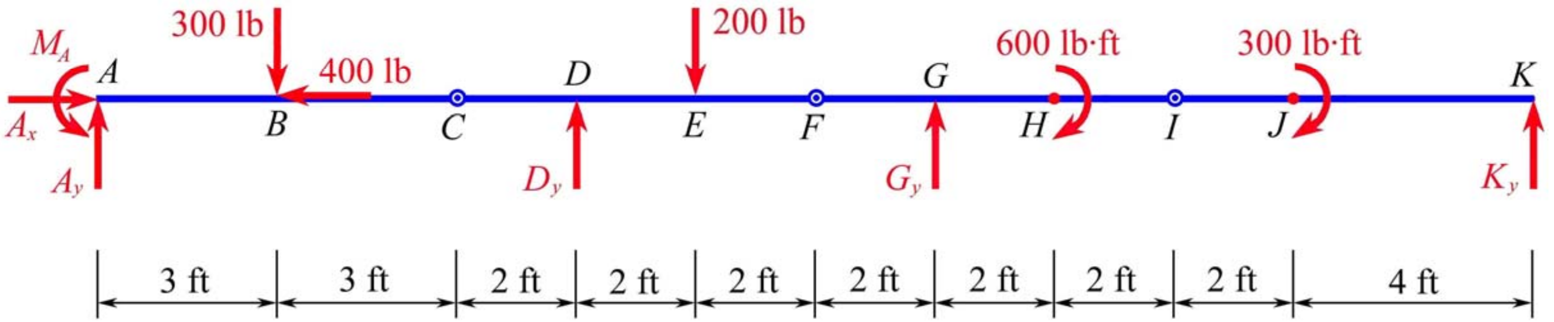
$$M_A = 2100 \qquad \mathbf{M}_A = 2100 \text{ lb}\cdot\text{ft} \circlearrowleft$$

Example 4. Determine the vertical reaction force $\mathbf{A}_y$ at the fixed support *A* of the combined beam as shown.

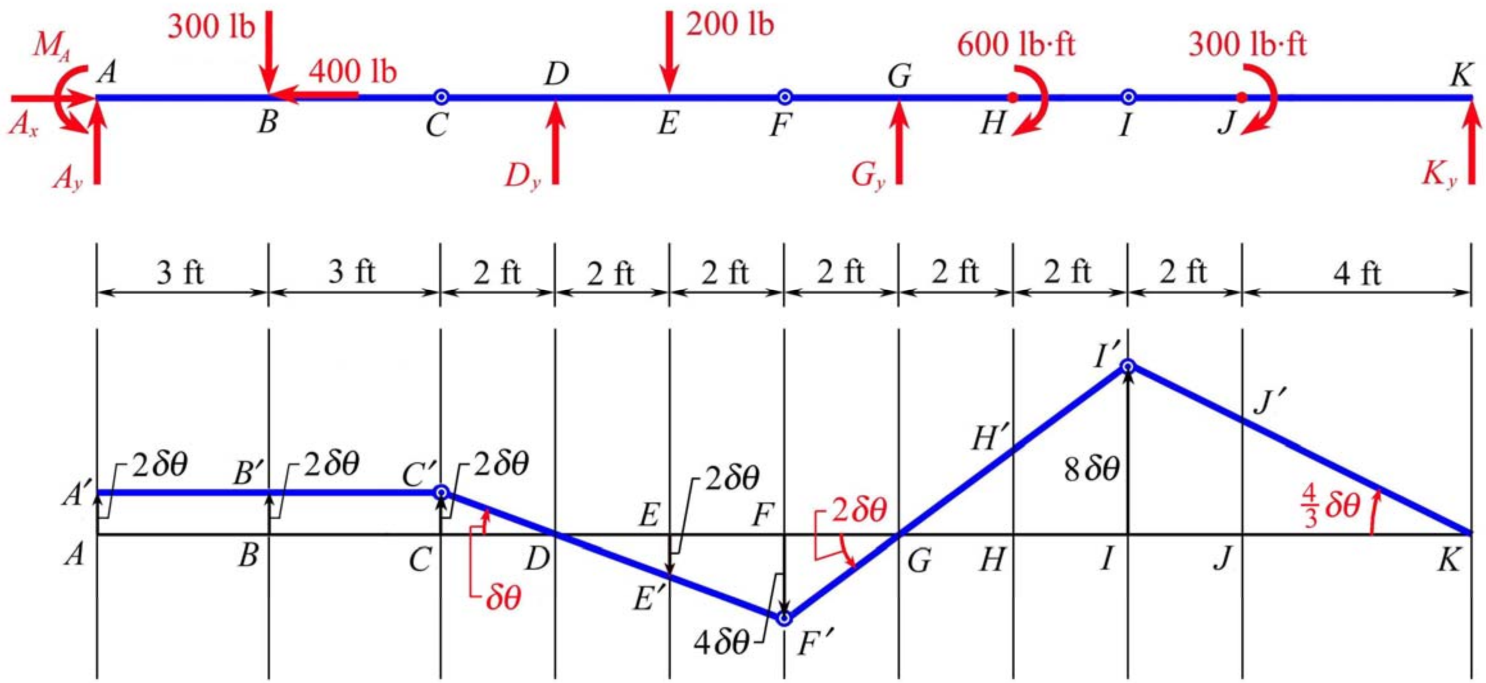


Solution.

Step 1: *FBD*



Step 2: *VDD* to involve A_y in δU (*FBD* repeated here for reference only)



Step 3: $\delta U = 0$:

$$A_y(2\,\delta\theta) + 300(-2\,\delta\theta) + 200(2\,\delta\theta) + 600(-2\,\delta\theta) + 300\left(\tfrac{4}{3}\delta\theta\right) = 0$$

$$A_y = 500$$

$$\mathbf{A}_y = 500 \text{ lb} \uparrow$$

Concluding Remarks

- Work ≠ Energy.
- Work is energy in transition, a boundary phenomenon.
- Virtual work is work done on a body undergoing virtual displacement.
- In a nut shell, the **virtual work method** in Statics consists of *three major steps* and *one guiding strategy*.
- The **three major steps** are: (*a*) draw the *FBD* of the system, (*b*) draw the *VDD* of the system with a guiding strategy, and (*c*) set $\delta U = 0$ to solve for the specified unknown.
- The **guiding strategy** in drawing the *VDD* is to give the free body a *compatible virtual displacement* in such a way that the *one* specified unknown, but *no other unknowns*, will be involved in the virtual work done.
- Virtual work method is truly a powerful method in mechanics.

Questions?