Learning the Virtual Work Method in Statics: What Is a Compatible Virtual Displacement?

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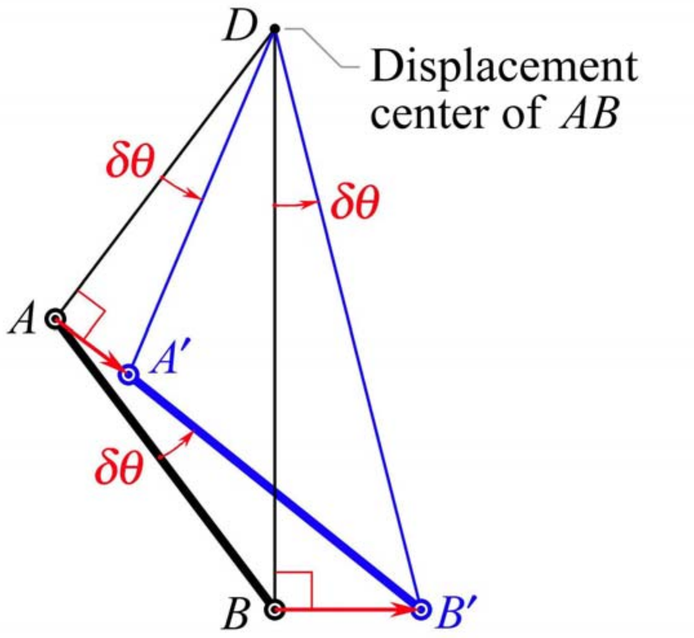
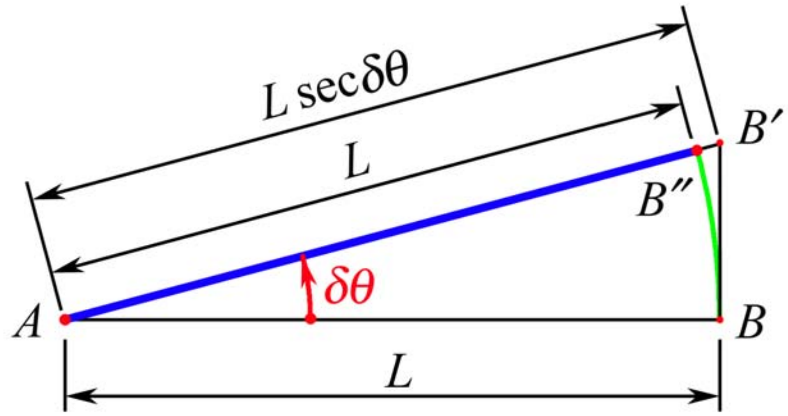
Why "virtual work method" in Statics?

- **Virtual work method** is a standing topic contained in most textbooks for Statics.
- This method and the traditional method *equally* require and emphasize the **drawing of *FBD*** in solving problems.
- The initial difficulty in learning it is *surmountable* with the understanding of some **key concepts** & a **right approach**.
- Virtual work method is usually covered in Statics at the *discretion* of instructors to **enrich the learning** of students.
- This paper is intended to share with fellow mechanics educators the **teaching** of virtual work method.

Key concepts: displacement center, compatible virtual displacement, & rigid-body virtual displacement





$$\overline{B''B'} \approx \frac{L}{2}(\delta\theta)^2 \to 0$$

A body undergoes a **compatible virtual displacement** from position *AB* to position *A′B′*, where the **displacement center** is at ***D***.

A body undergoes a **rigid-body virtual displacement** from *AB* to *AB″*, and a **compatible virtual displacement** from *AB* to *AB′*. The *displacement center* is at ***A***.

Work of a force: $U_{1\to 2} = \mathbf{F} \cdot \mathbf{q} = F q_{\|}$

Work of a moment: $U_{1\to 2} = M(\Delta\theta)$

Radian measure formula: $\delta s = r\,\delta\theta$

Virtual work: It is the work done by a force or moment on a body during a *virtual displacement* of the body.

Principle of virtual work:

If a body is in equilibrium, the total virtual work δU of the external force system acting on its free body during any *compatible virtual displacement* of its free body is equal to zero; i.e.,

$$\delta U = 0$$

A right approach to virtual work method:

Three major steps:

Step 1: Draw the free-body diagram (*FBD*).

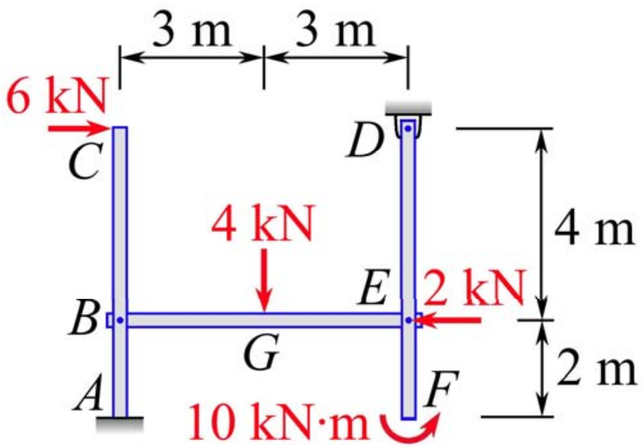
Step 2: Draw the virtual-displacement diagram (*VDD*) with a strategy.

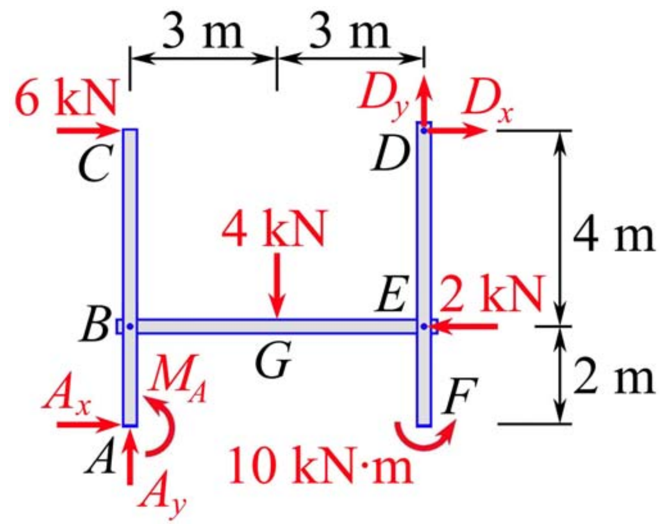
Step 3: Set to zero the total virtual work done.

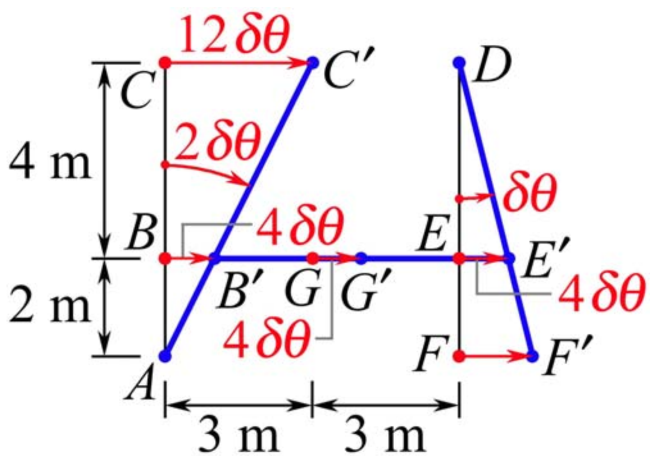
One strategy:

In step 2, give the free body a *compatible virtual displacement* in such a way that the *one* specified unknown, but *no other unknowns*, will be involved in the total virtual work done.

Example 1. Determine *only* the reaction moment $\mathbf{M}_A$ at the fixed support A of the frame loaded as shown.





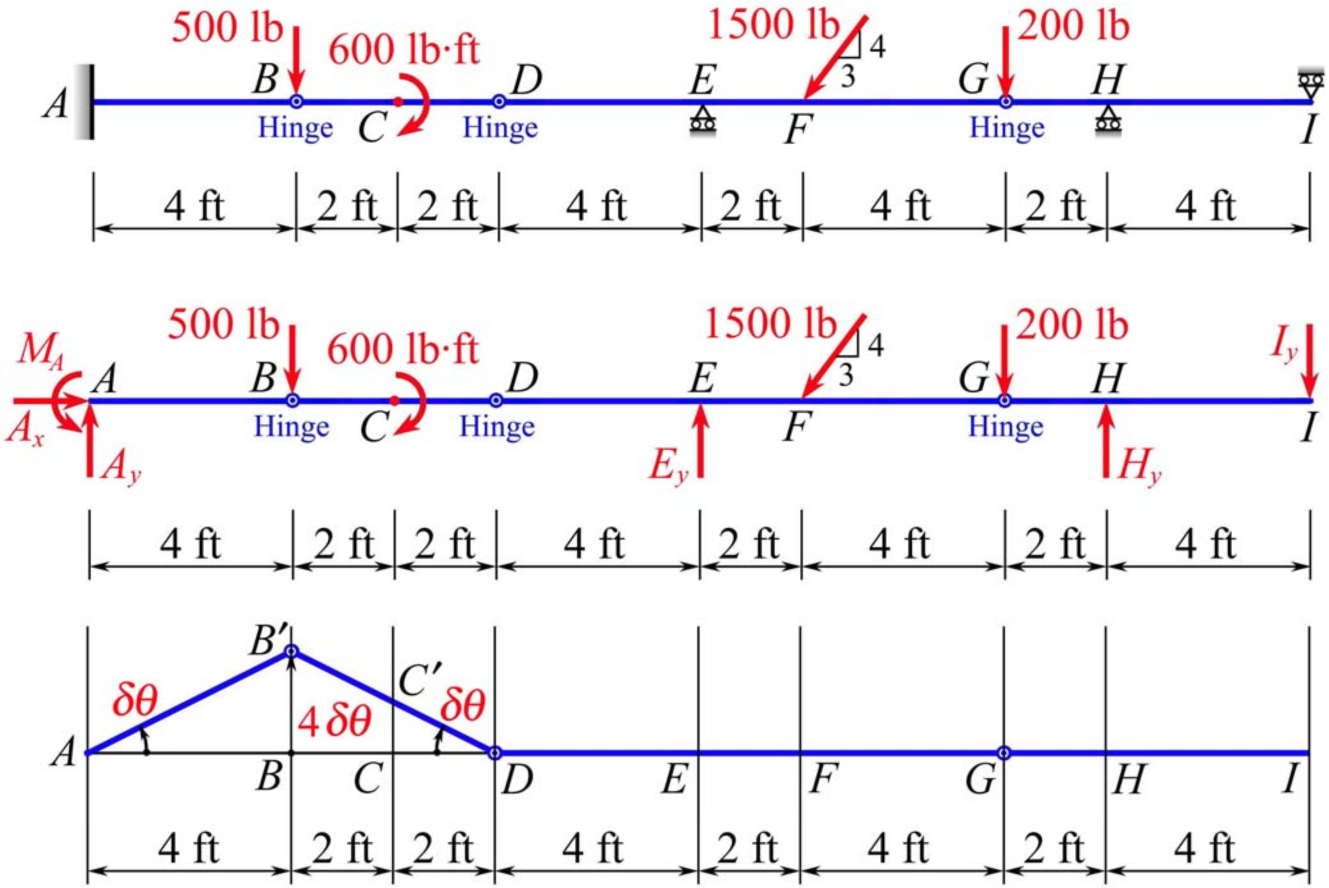


$\delta U = 0$:

$$M_A(-2\,\delta\theta) + 6(12\,\delta\theta)$$
$$+2(-4\,\delta\theta) + 10\,\delta\theta = 0$$

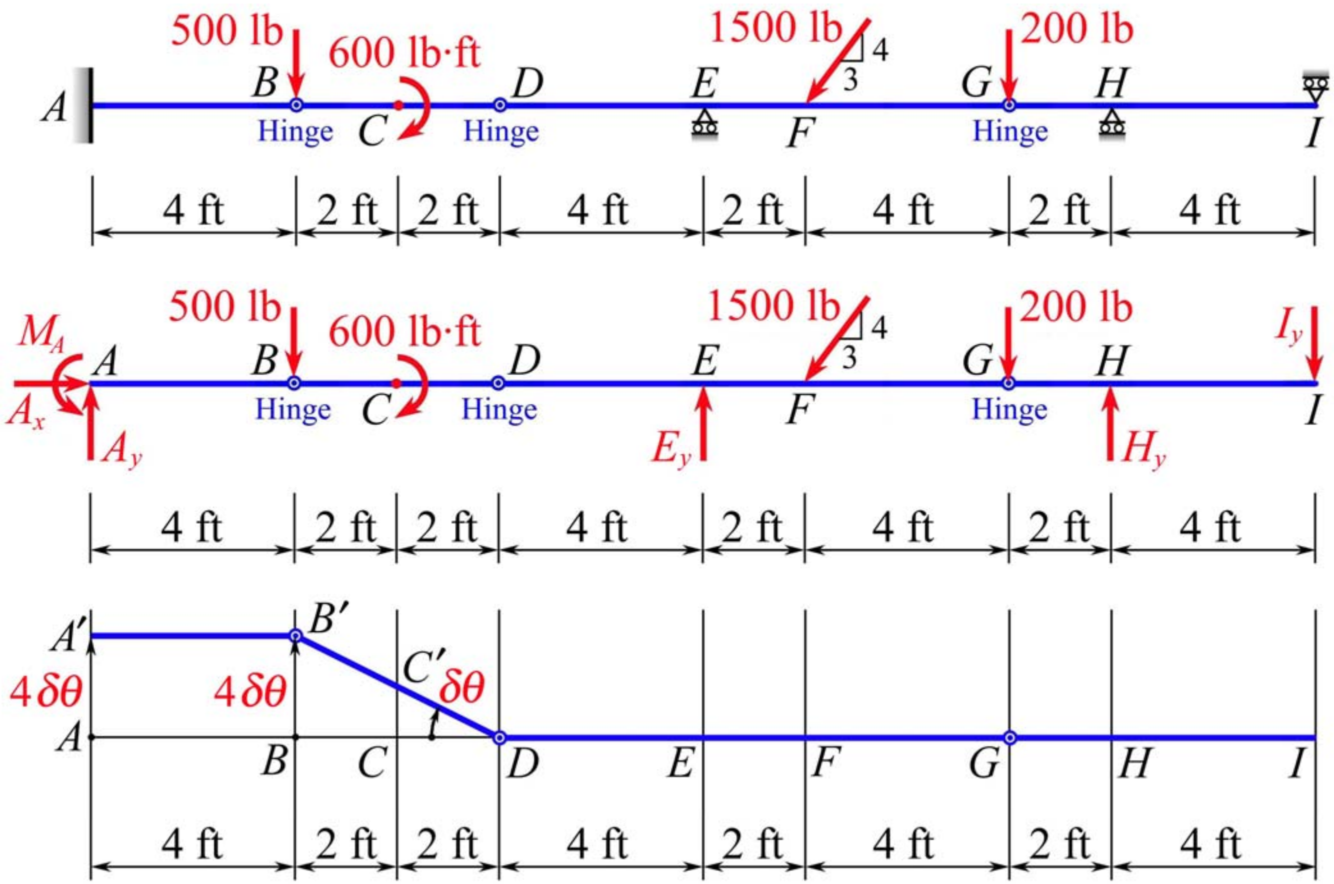
$M_A = 37$ $\quad$ $\mathbf{M}_A = 37 \text{ kN·m}$ ↺

Example 2. Determine *only* the reaction moment $\mathbf{M}_A$ at the fixed support A of the beam loaded as shown.



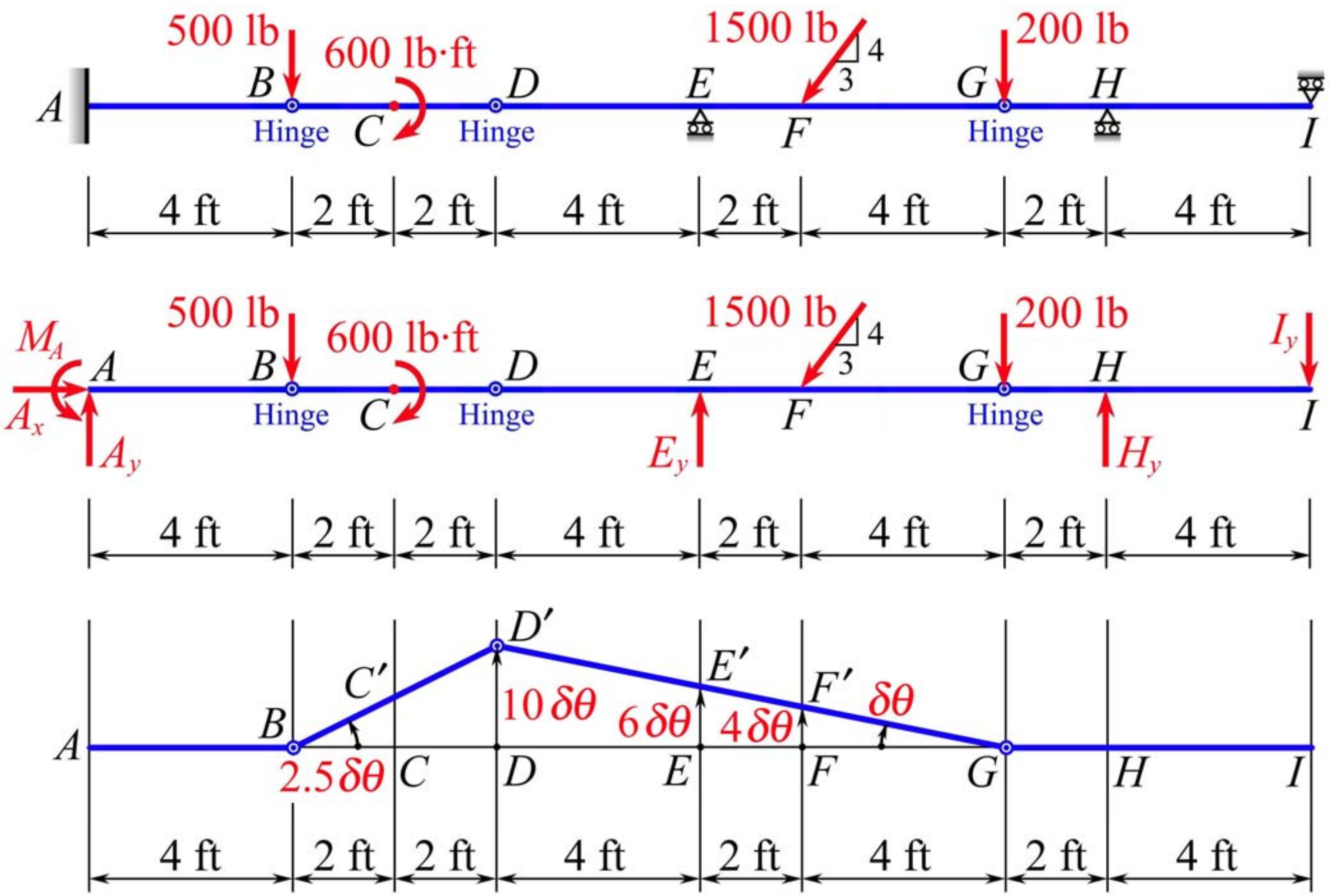
$\delta U = 0$: $\quad M_A(\delta\theta) + 500(-4\,\delta\theta) + 600(\delta\theta) = 0 \qquad M_A = 1400 \qquad \mathbf{M}_A = 1400\ \text{lb}\cdot\text{ft}\ \circlearrowleft$

Example 3. Determine *only* the vertical reaction force $\mathbf{A}_y$ at the fixed support A of the beam loaded as shown.



$$\delta U = 0: \quad A_y(4\,\delta\theta) + 500(-4\,\delta\theta) + 600(\delta\theta) = 0 \qquad A_y = 350 \qquad \mathbf{A}_y = 350 \text{ lb } \uparrow$$

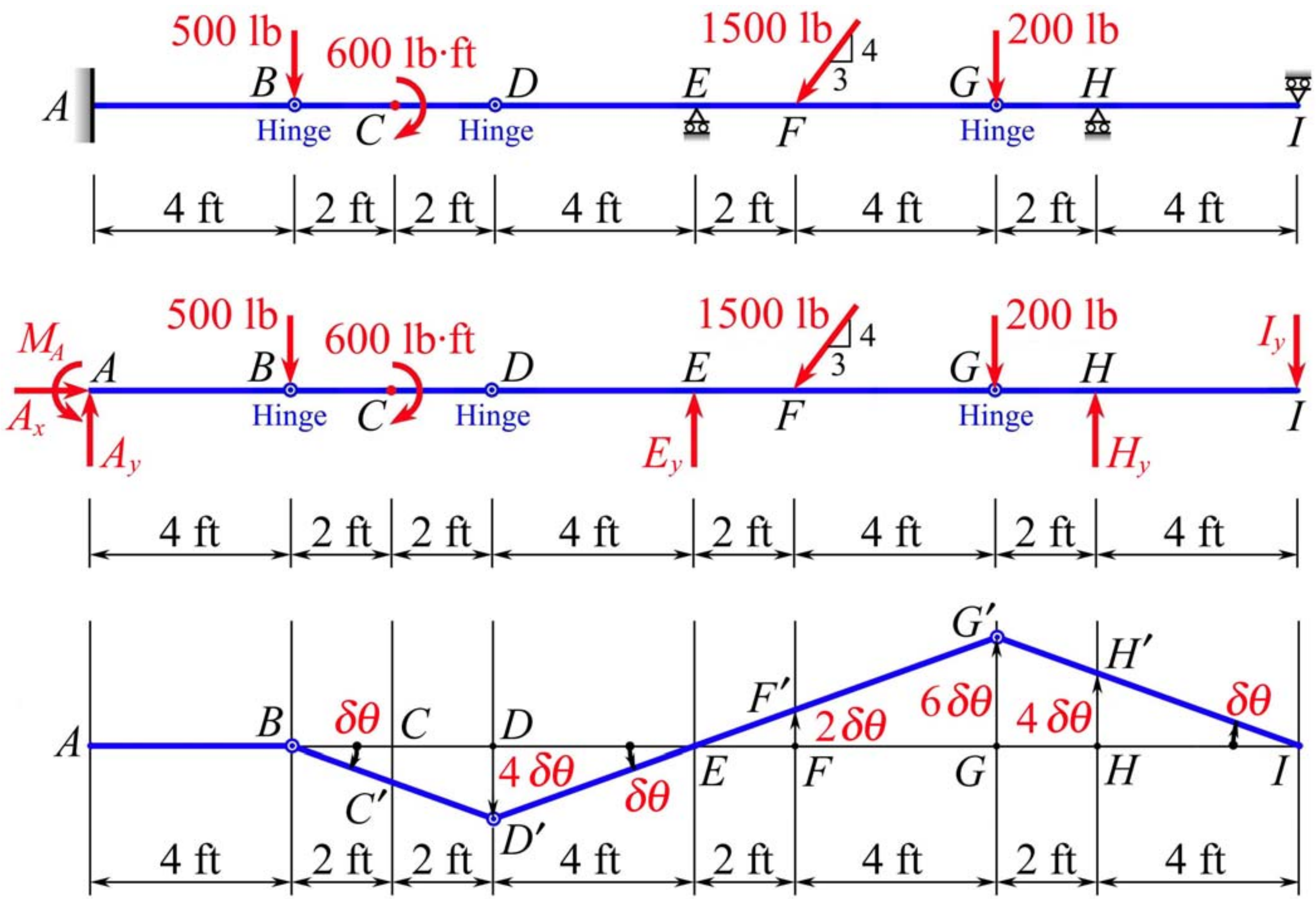
Example 4. Determine *only* the vertical reaction force $\mathbf{E}_y$ at the roller support E of the beam loaded as shown.



$\delta U = 0$: $\quad 600(-2.5\delta\theta) + E_y(6\delta\theta) + \frac{4}{5}(1500)(-4\delta\theta) = 0 \qquad E_y = 1050$

$\mathbf{E}_y = 1050 \text{ lb} \uparrow$

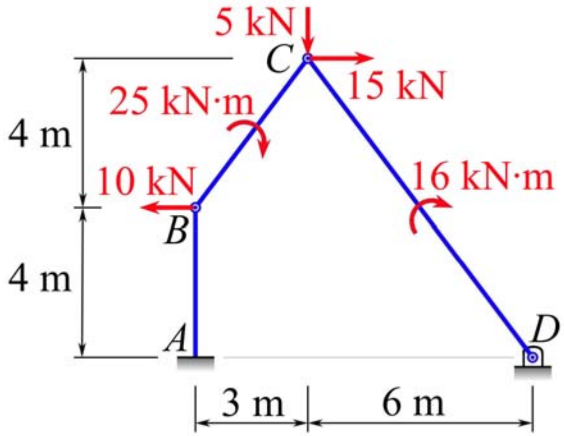
Example 5. Determine *only* the vertical reaction force $\mathbf{H}_y$ at the roller support *H* of the beam loaded as shown.

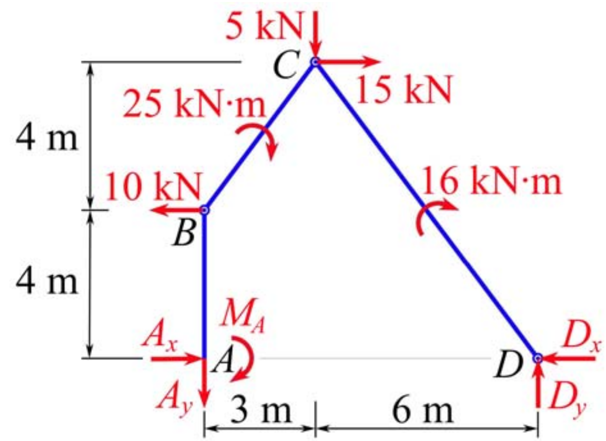


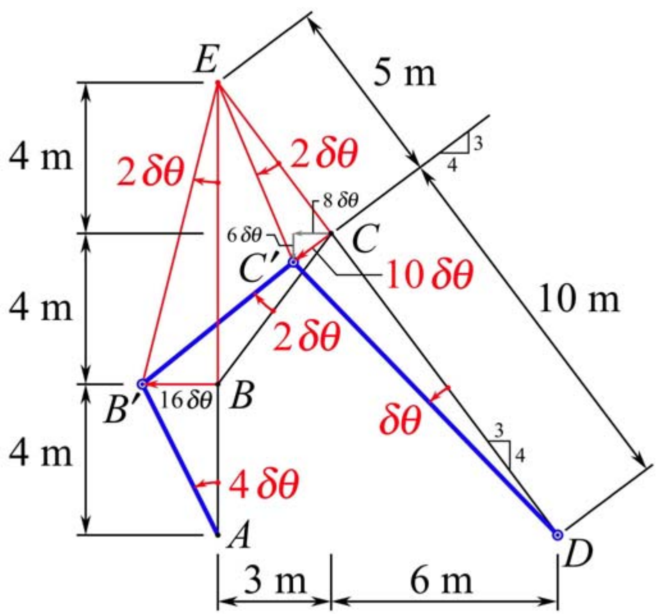
$$\delta U = 0: \quad 600(\delta\theta) + \frac{4}{5}(1500)(-2\,\delta\theta) + 200(-6\,\delta\theta) + H_y(4\,\delta\theta) = 0 \quad H_y = 750$$

$\mathbf{H}_y = 750 \text{ lb} \uparrow$

Example 6. Determine *only* the reaction moment $\mathbf{M}_A$ at the fixed support A of the frame loaded as shown.





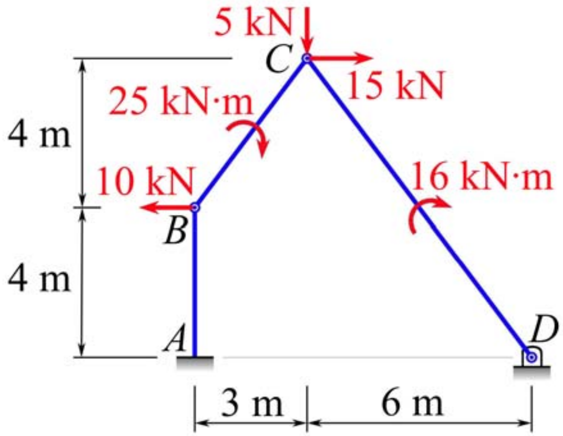


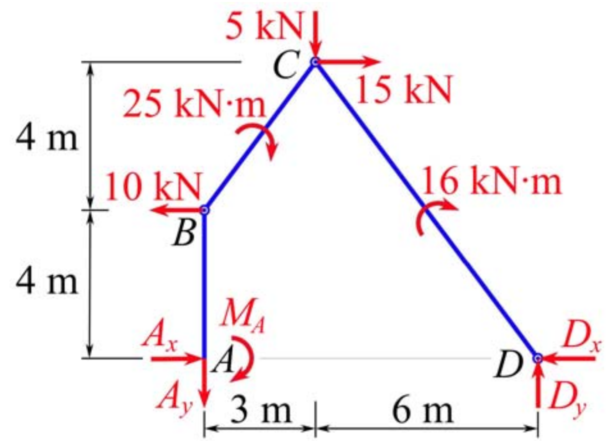
$\delta U = 0$:

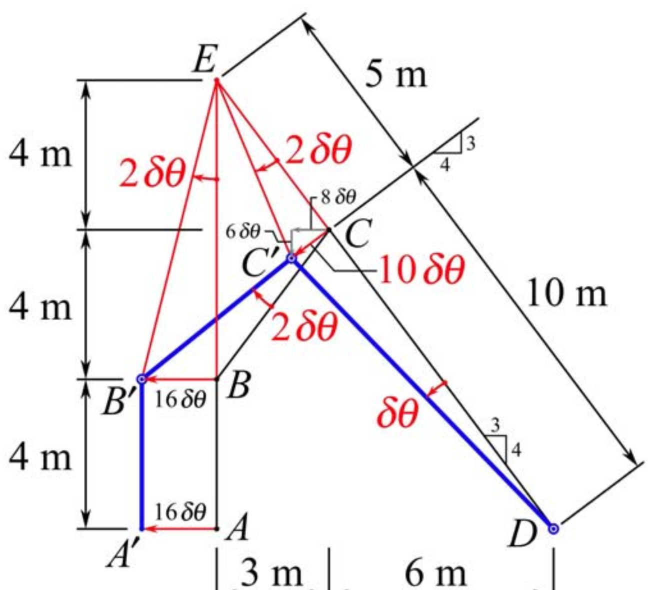
$$M_A(-4\,\delta\theta) + 10(16\,\delta\theta)$$
$$+25(2\,\delta\theta) + 15(-8\,\delta\theta)$$
$$+5(6\,\delta\theta) + 16(-\delta\theta) = 0$$

$M_A = 26$ $\quad \mathbf{M}_A = 26\ \text{kN·m}\ \circlearrowleft$

Example 7. Determine *only* the horizontal reaction force $\mathbf{A}_x$ at the fixed support A of the frame loaded as shown.







$\delta U = 0$:

$$A_x(-16\,\delta\theta) + 10(16\,\delta\theta)$$
$$+25(2\,\delta\theta) + 15(-8\,\delta\theta)$$
$$+5(6\,\delta\theta) + 16(-\delta\theta) = 0$$

$A_x = 6.5$ $\qquad$ $\mathbf{A}_x = 6.5\text{ kN} \rightarrow$

Concluding Remarks

- The key concepts in virtual work method include: work of a force, work of a moment, displacement center, *compatible virtual displacement*, and radian measure formula.
- The virtual work method in Statics consists of **three major steps**: (*a*) draw the *FBD*, (*b*) draw the *VDD* with a strategy, and (*c*) set $\delta U = 0$ to solve for the unknown.
- The **strategy** in drawing the *VDD* is to give the free body a *compatible virtual displacement* in such a way that the *one* specified unknown, but *no other unknowns*, will be involved in the total virtual work done.
- George Bernard Shaw once said, "You see things; and you say, 'Why?' But I dream things that never were; and I say, 'Why not?'"

Questions?