Teaching Deflections of Beams: Advantages of Method of Model Formulas versus Method of Integration

Ing-Chang Jong, William T. Springer, Rick J. Couvillion
University of Arkansas, Fayetteville, AR 72701

Abstract

The *method of model formulas* is a new method for solving statically indeterminate reactions and deflections of elastic beams. Since its publication in a recent issue of the *IJEE*,[1] many instructors of Mechanics of Materials have considerable interest in knowing an effective way for teaching this method to enrich students' study and their set of skills in determining beam reactions and deflections. Moreover, people are interested in seeing demonstrations showing any advantage of this method over the traditional methods. This paper is aimed at (*a*) providing comparisons of this new method versus the traditional *method of integration* via several head-to-head contrasting solutions of same problems, and (*b*) proposing a set of steps for use to effectively introduce and teach this new method to students. It is a considered opinion that the *method of model formulas* be taught to students after having taught them one or more of the traditional methods.

I. Introduction

Beams are longitudinal members subjected to transverse loads. Students usually first learn the design of beams for strength. Then they learn the determination of deflections of beams under a variety of loads. Traditional methods used in determining statically indeterminate reactions and deflections of elastic beams include:[2-12] method of integration (*with* or *without* use of singularity functions), method of superposition, method using moment-area theorems, method of conjugate beam, method using Castigliano's theorem, and method of segments.

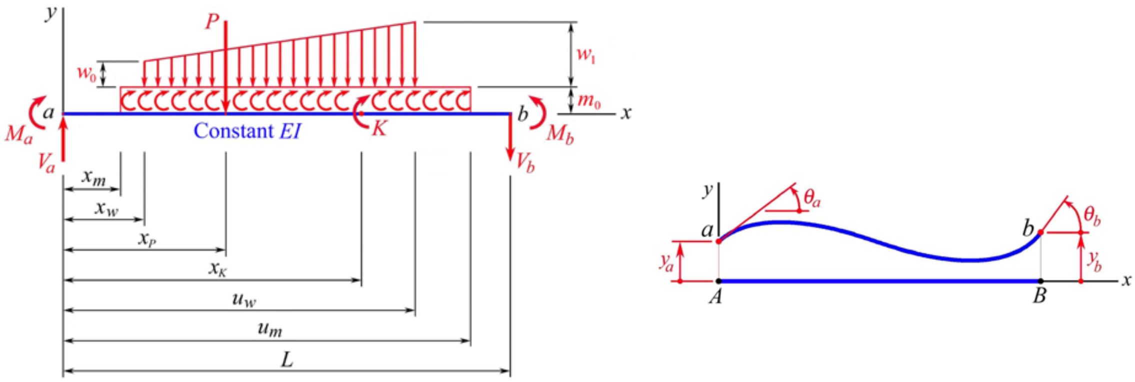
The *method of model formulas*[1] is a newly propounded method. Beginning with an elastic beam under a preset general loading, a set of *four* model formulas are derived and established for use in this new method. These formulas are expressed in terms of the following:

(*a*) flexural rigidity of the beam;
(*b*) slopes, deflections, shear forces, and bending moments at both ends of the beam;
(*c*) typical applied loads (concentrated force, concentrated moment, linearly distributed force, and uniformly distributed moment) somewhere on the beam.

For starters, one must know that a working **proficiency** in the rudiments of *singularity functions* is a **prerequisite** to using the *method of model formulas*. To benefit a wider readership, which may have different specialties in mechanics, and to avoid or minimize any possible misunderstanding, this paper includes summaries of the rudiments of singularity functions and the sign conventions for beams. Readers, who are familiar with these topics, may skip the summaries. An excerpt from the *method of model formulas* is needed and shown in Fig. 1, courtesy of *IJEE*.[1]

Excerpt from the **Method of Model Formulas**

Courtesy: *Int. J. Engng. Ed.*, Vol. 25, No. 1, pp. 65-74, 2009



(a) *(b)*

Positive directions of forces, moments, slopes, and deflections

$$\begin{aligned} y' = {} & \theta_a + \frac{V_a}{2EI}x^2 + \frac{M_a}{EI}x - \frac{P}{2EI}\langle x-x_P\rangle^2 + \frac{K}{EI}\langle x-x_K\rangle^1 - \frac{w_0}{6EI}\langle x-x_w\rangle^3 \\ & - \frac{w_1-w_0}{24EI(u_w-x_w)}\langle x-x_w\rangle^4 + \frac{w_1}{6EI}\langle x-u_w\rangle^3 + \frac{w_1-w_0}{24EI(u_w-x_w)}\langle x-u_w\rangle^4 \\ & + \frac{m_0}{2EI}\langle x-x_m\rangle^2 - \frac{m_0}{2EI}\langle x-u_m\rangle^2 \end{aligned} \tag{1}$$

$$\begin{aligned} y = {} & y_a + \theta_a x + \frac{V_a}{6EI}x^3 + \frac{M_a}{2EI}x^2 - \frac{P}{6EI}\langle x-x_P\rangle^3 + \frac{K}{2EI}\langle x-x_K\rangle^2 - \frac{w_0}{24EI}\langle x-x_w\rangle^4 \\ & - \frac{w_1-w_0}{120EI(u_w-x_w)}\langle x-x_w\rangle^5 + \frac{w_1}{24EI}\langle x-u_w\rangle^4 + \frac{w_1-w_0}{120EI(u_w-x_w)}\langle x-u_w\rangle^5 \\ & + \frac{m_0}{6EI}\langle x-x_m\rangle^3 - \frac{m_0}{6EI}\langle x-u_m\rangle^3 \end{aligned} \tag{2}$$

$$\begin{aligned} \theta_b = {} & \theta_a + \frac{V_a L^2}{2EI} + \frac{M_a L}{EI} - \frac{P}{2EI}(L-x_P)^2 + \frac{K}{EI}(L-x_K) - \frac{w_0}{6EI}(L-x_w)^3 \\ & - \frac{w_1-w_0}{24EI(u_w-x_w)}(L-x_w)^4 + \frac{w_1}{6EI}(L-u_w)^3 + \frac{w_1-w_0}{24EI(u_w-x_w)}(L-u_w)^4 \\ & + \frac{m_0}{2EI}(L-x_m)^2 - \frac{m_0}{2EI}(L-u_m)^2 \end{aligned} \tag{3}$$

$$\begin{aligned} y_b = {} & y_a + \theta_a L + \frac{V_a L^3}{6EI} + \frac{M_a L^2}{2EI} - \frac{P}{6EI}(L-x_P)^3 + \frac{K}{2EI}(L-x_K)^2 - \frac{w_0}{24EI}(L-x_w)^4 \\ & - \frac{w_1-w_0}{120EI(u_w-x_w)}(L-x_w)^5 + \frac{w_1}{24EI}(L-u_w)^4 + \frac{w_1-w_0}{120EI(u_w-x_w)}(L-u_w)^5 \\ & + \frac{m_0}{6EI}(L-x_m)^3 - \frac{m_0}{6EI}(L-u_m)^3 \end{aligned} \tag{4}$$

Fig. 1. Loading, deflections, and formulas in the *Method of Model Formulas* for beams

■ Summary of rudiments of singularity functions:

Notice that the argument of a singularity function is enclosed by angle brackets (i.e., < >). The argument of a regular function continues to be enclosed by parentheses [i.e., ( )]. The rudiments of singularity functions include the following:[8,9]

$$< x-a >^n = (x-a)^n \quad \text{if} \quad x-a \geq 0 \quad \text{and} \quad n > 0 \tag{5}$$

$$< x-a >^n = 1 \quad \text{if} \quad x-a \geq 0 \quad \text{and} \quad n = 0 \tag{6}$$

$$< x-a >^n = 0 \quad \text{if} \quad x-a < 0 \quad \text{or} \quad n < 0 \tag{7}$$

$$\int_{-\infty}^{x} < x-a >^n dx = \frac{1}{n+1} < x-a >^{n+1} \quad \text{if} \quad n > 0 \tag{8}$$

$$\int_{-\infty}^{x} < x-a >^n dx = < x-a >^{n+1} \quad \text{if} \quad n \leq 0 \tag{9}$$

$$\frac{d}{dx} < x-a >^n = n < x-a >^{n-1} \quad \text{if} \quad n > 0 \tag{10}$$

$$\frac{d}{dx} < x-a >^n = < x-a >^{n-1} \quad \text{if} \quad n \leq 0 \tag{11}$$

Equations (6) and (7) imply that, in using singularity functions for beams, we take

$$b^0 = 1 \quad \text{for} \quad b \geq 0 \tag{12}$$

$$b^0 = 0 \quad \text{for} \quad b < 0 \tag{13}$$

■ Summary of sign conventions for beams:

In the *method of model formulas*, the adopted sign conventions for various model loadings on the beam and for deflections of the beam with a constant flexural rigidity *EI* are illustrated in Fig. 1. Notice the following key points:

- A *shear force* is *positive* if it acts upward on the left (or downward on the right) face of the beam element [e.g., $\mathbf{V}_a$ at the left end *a*, and $\mathbf{V}_b$ at the right end *b* in Fig. 1(*a*)].
- At ends of the beam, a *moment* is *positive* if it tends to cause compression in the top fiber of the beam [e.g., $\mathbf{M}_a$ at the left end *a*, and $\mathbf{M}_b$ at the right end *b* in Fig. 1(*a*)].
- If not at ends of the beam, a *moment* is *positive* if it tends to cause compression in the top fiber of the beam just to the right of the position where it acts [e.g., the concentrated moment $\mathbf{K} = K \circlearrowleft$ and the uniformly distributed moment with intensity m_0 in Fig. 1(*a*)].
- A *concentrated force* or a *distributed force* applied to the beam is *positive* if it is directed downward [e.g., the concentrated force $\mathbf{P} = P \downarrow$, the linearly distributed force with intensity w_0 on the left side and intensity w_1 on the right side in Fig. 1(*a*), where the distribution becomes uniform if $w_0 = w_1$].

The slopes and deflections of a beam displaced from *AB* to *ab* are shown in Fig. 1(*b*). Note that

- A *positive slope* is a counterclockwise angular displacement [e.g., θ_a and θ_b in Fig. 1(*b*)].
- A *positive deflection* is an upward linear displacement [e.g., y_a and y_b in Fig. 1(*b*)].

II. Teaching and Learning a New Method via Contrast between Solutions

Equations (1) through (4) are related to the beam and loading shown in Fig. 1; they are the *model formulas* in the new method. Their derivation (*not* a main concern in this paper) can be found in the paper that propounded the **method of model formulas.**[1] Note that L in the model formulas in Eqs. (1) through (4) is a *parameter* representing the *total length* of the beam. In other words, L is to be replaced by the *total length* of the beam segment, to which the model formulas are applied. Statically indeterminate reactions as well as slopes and deflections of beams can, of course, be solved. A beam needs to be divided into segments for analysis only if (*a*) it is a combined beam (e.g., a *Gerber beam*) having discontinuities in slope at hinge connections between segments, and (*b*) it contains segments with different flexural rigidities (e.g., a stepped beam). Having learned an additional efficacious method, students' study and set of skills are enriched.

Mechanics is mostly a deductive science, but learning is mostly an inductive process. For the purposes of **teaching** and **learning**, all examples will be **first** solved by the traditional *method of integration* (**MoI**) — *with* use of singularity functions — **then** solved again by the *method of model formulas* (**MoMF**). As usual, the loading function, shear force, bending moment, slope, and deflection of the beam are denoted by the symbols q, V, M, y', and y, respectively.

Example 1. A simply supported beam AD with constant flexural rigidity EI and length L is acted on by a concentrated force $P\downarrow$ at B and a concentrated moment PL ↺ at C as shown in Fig. 2. Determine (*a*) the slopes θ_A and θ_D at A and D, respectively; (*b*) the deflection y_B at B.

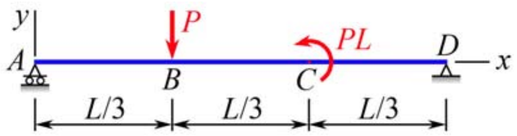


Fig. 2. Simply supported beam AD carrying concentrated loads

Solution. The beam is in static equilibrium. Its free-body diagram is shown in Fig. 3.

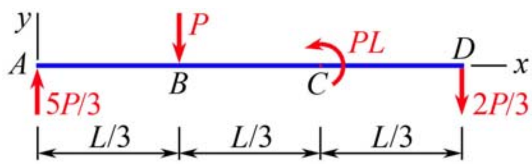


Fig. 3. Free-body diagram of the simply supported beam AD

● **Using MoI:** Using the symbols defined earlier and applying the *method of integration* (*with* use of singularity functions) to this beam, we write

$$q = \frac{5P}{3} < x >^{-1} - P < x - \frac{L}{3} >^{-1} - PL < x - \frac{2L}{3} >^{-2}$$

$$V = \frac{5P}{3} < x >^{0} - P < x - \frac{L}{3} >^{0} - PL < x - \frac{2L}{3} >^{-1}$$

$$M = EIy'' = \frac{5P}{3} < x >^{1} - P < x - \frac{L}{3} >^{1} - PL < x - \frac{2L}{3} >^{0}$$

$$EIy' = \frac{5P}{6} < x >^2 - \frac{P}{2} < x - \frac{L}{3} >^2 - PL < x - \frac{2L}{3} >^1 + C_1$$
$$EIy = \frac{5P}{18} < x >^3 - \frac{P}{6} < x - \frac{L}{3} >^3 - \frac{PL}{2} < x - \frac{2L}{3} >^2 + C_1 x + C_2$$

The *boundary conditions* of this beam reveal that $y(0) = 0$ at *A* and $y(L) = 0$ at *D*. Imposing these *two* conditions, respectively, we write

$$0 = C_2$$
$$0 = \frac{5P}{18}(L)^3 - \frac{P}{6}\left(\frac{2L}{3}\right)^3 - \frac{PL}{2}\left(\frac{L}{3}\right)^2 + C_1 L$$

These *two* simultaneous equations yield

$$C_1 = -\frac{14PL^2}{81} \qquad C_2 = 0$$

Using these values and the foregoing equations for EIy' and EIy, we write

$$\theta_A = y'\big|_{x=0} = \frac{C_1}{EI} = -\frac{14PL^2}{81EI} \qquad \theta_D = y'\big|_{x=L} = \frac{5P}{6EI}(L)^2 - \frac{P}{2EI}\left(\frac{2L}{3}\right)^2 - \frac{PL}{EI}\left(\frac{L}{3}\right) + \frac{C_1}{EI} = \frac{17PL^2}{162EI}$$

$$y_B = y\big|_{x=L/3} = \frac{5P}{18EI}\left(\frac{L}{3}\right)^3 + \frac{C_1}{EI}\left(\frac{L}{3}\right) = -\frac{23PL^3}{486EI}$$

We report that

$$\boxed{\theta_A = \frac{14PL^2}{81EI} \circlearrowright} \qquad \boxed{\theta_D = \frac{17PL^2}{162EI} \circlearrowleft} \qquad \boxed{y_B = \frac{23PL^3}{486EI} \downarrow}$$

● **Using MoMF:** In applying the *method of model formulas* to this beam, we must adhere to the sign conventions as illustrated in Fig. 1. At the left end *A*, the moment M_A is 0, the shear force V_A is 5*P*/3, the deflection y_A is 0, but the slope θ_A is unknown. At the right end *D*, the deflection y_D is 0, but the slope θ_D is unknown. Note in the model formulas that we have $x_P = L/3$ for the concentrated force $P\downarrow$ at *B* and $x_K = 2L/3$ for the concentrated moment $PL \circlearrowleft$ at *C*. Applying the model formulas in Eqs. (3) and (4), successively, to this beam *AD*, we write

$$\theta_D = \theta_A + \frac{(5P/3)L^2}{2EI} + 0 - \frac{P}{2EI}\left(L - \frac{L}{3}\right)^2 + \frac{-PL}{EI}\left(L - \frac{2L}{3}\right) - 0 - 0 + 0 + 0 + 0 - 0$$

$$0 = 0 + \theta_A L + \frac{(5P/3)L^3}{6EI} + 0 - \frac{P}{6EI}\left(L - \frac{L}{3}\right)^3 + \frac{-PL}{2EI}\left(L - \frac{2L}{3}\right)^2 - 0 - 0 + 0 + 0 + 0 - 0$$

These *two* simultaneous equations yield

$$\theta_A = -\frac{14PL^2}{81EI} \qquad \theta_D = \frac{17PL^2}{162EI}$$

Using the value of θ_A and applying the model formula in Eq. (2), we write

$$y_B = y\big|_{x=L/3} = 0 + \theta_A\left(\frac{L}{3}\right) + \frac{5P/3}{6EI}\left(\frac{L}{3}\right)^3 + 0 - 0 + 0 - 0 - 0 + 0 + 0 + 0 - 0 = -\frac{23PL^3}{486EI}$$

We report that

$$\theta_A = \frac{14PL^2}{81EI} \circlearrowleft \qquad \theta_D = \frac{17PL^2}{162EI} \circlearrowright \qquad y_B = \frac{23PL^3}{486EI} \downarrow$$

Remark. We observe that both the *method of integration* (*with* use of singularity functions) and the *method of model formulas* yield the same solutions, as expected. In fact, the solution by the **MoMF** looks more direct than that by the **MoI**. Furthermore, if singularity functions were *not* used in the **MoI**, the solution would require division of the beam into multiple segments (such as *AB*, *BC*, and *CD*), and much more effort in algebraic work in the solution would be involved. In Examples 2 through 5, readers may observe similar features.

Example 2. A cantilever beam *AC* with constant flexural rigidity *EI* and length *L* is loaded with a distributed load of intensity *w* in segment *AB* as shown in Fig. 4. Determine (*a*) the slope θ_A and deflection y_A at *A*, (*b*) the slope θ_B and deflection y_B at *B*.

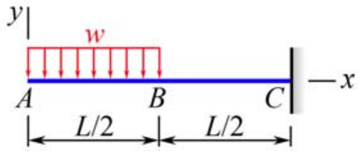


Fig. 4. Cantilever beam *AC* loaded with a distributed load

Solution. The beam is in static equilibrium. Its free-body diagram is shown in Fig. 5.

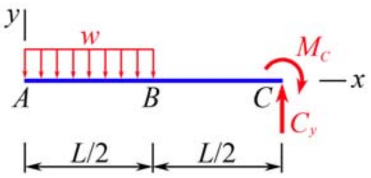


Fig. 5. Free-body diagram of the cantilever beam *AC*

● **Using MoI:** Applying the *method of integration* to this beam, we write

$$q = -w < x >^0 + w < x - \frac{L}{2} >^0$$

$$V = -w < x >^1 + w < x - \frac{L}{2} >^1$$

$$M = EIy'' = -\frac{w}{2} < x >^2 + \frac{w}{2} < x - \frac{L}{2} >^2$$

$$EIy' = -\frac{w}{6} < x >^3 + \frac{w}{6} < x - \frac{L}{2} >^3 + C_1$$

$$EIy = -\frac{w}{24} < x >^4 + \frac{w}{24} < x - \frac{L}{2} >^4 + C_1 x + C_2$$

The *boundary conditions* of this beam reveal that $y'(L) = 0$ and $y(L) = 0$ at *C*. Imposing these *two* conditions, respectively, we write

$$0 = -\frac{w}{6}L^3 + \frac{w}{6}\left(\frac{L}{2}\right)^3 + C_1$$

$$0 = -\frac{w}{24}L^4 + \frac{w}{24}\left(\frac{L}{2}\right)^4 + C_1 L + C_2$$

These *two* simultaneous equations yield

$$C_1 = \frac{7wL^3}{48} \qquad C_2 = -\frac{41wL^4}{384}$$

Using these values and the foregoing equations for EIy' and EIy, we write

$$\theta_A = y'\big|_{x=0} = \frac{C_1}{EI} = \frac{7wL^3}{48EI} \qquad y_A = y\big|_{x=0} = \frac{C_2}{EI} = -\frac{41wL^4}{384EI}$$

$$\theta_B = y'\big|_{x=L/2} = -\frac{w}{6EI}\left(\frac{L}{2}\right)^3 + \frac{C_1}{EI} = \frac{wL^3}{8EI}$$

$$y_B = y\big|_{x=L/2} = -\frac{w}{24EI}\left(\frac{L}{2}\right)^4 + \frac{C_1}{EI}\left(\frac{L}{2}\right) + \frac{C_2}{EI} = -\frac{7wL^4}{192EI}$$

We report that

$$\boxed{\theta_A = \frac{7wL^3}{48EI} \circlearrowleft} \qquad \boxed{y_A = \frac{41wL^4}{384EI} \downarrow} \qquad \boxed{\theta_B = \frac{wL^3}{8EI} \circlearrowleft} \qquad \boxed{y_B = \frac{7wL^4}{192EI} \downarrow}$$

● **Using MoMF:** Let the *method of model formulas* be now applied to this beam. The shear force V_A and bending moment M_A at the free end *A*, as well as the slope θ_C and deflection y_C at the fixed end *C*, are all zero. Noting that $x_w = 0$ and $u_w = L/2$, we apply the model formulas in Eqs. (3) and (4) to the entire beam to write

$$0 = \theta_A + 0 + 0 - 0 + 0 - \frac{w}{6EI}L^3 - 0 + \frac{w}{6EI}\left(L - \frac{L}{2}\right)^3 + 0 + 0 - 0$$

$$0 = y_A + \theta_A L + 0 + 0 - 0 + 0 - \frac{w}{24EI}L^4 - 0 + \frac{w}{24EI}\left(L - \frac{L}{2}\right)^4 + 0 + 0 - 0$$

These *two* simultaneous equations yield

$$\theta_A = \frac{7wL^3}{48EI} \qquad y_A = -\frac{41wL^4}{384EI}$$

Using these values and applying the model formulas in Eqs. (1) and (2), respectively, we write

$$\theta_B = y'\big|_{x=L/2} = \theta_A + 0 + 0 - 0 + 0 - \frac{w}{6EI}\left(\frac{L}{2}\right)^3 - 0 + 0 + 0 + 0 - 0 = \frac{wL^3}{8EI}$$

$$y_B = y\big|_{x=L/2} = y_A + \theta_A\left(\frac{L}{2}\right) + 0 + 0 - 0 + 0 - \frac{w}{24EI}\left(\frac{L}{2}\right)^4 - 0 + 0 + 0 + 0 - 0 = -\frac{7wL^4}{192EI}$$

We report that

$$\boxed{\theta_A = \frac{7wL^3}{48EI} \circlearrowleft} \qquad \boxed{y_A = \frac{41wL^4}{384EI} \downarrow} \qquad \boxed{\theta_B = \frac{wL^3}{8EI} \circlearrowleft} \qquad \boxed{y_B = \frac{7wL^4}{192EI} \downarrow}$$

Example 3. A cantilever beam AC with constant flexural rigidity EI and total length $2L$ is propped at A and carries a concentrated moment M_0 ↺ at B as shown in Fig. 6. Determine (*a*) the vertical reaction force $\mathbf{A}_y$ and slope θ_A at A, (*b*) the slope θ_B and deflection y_B at B.

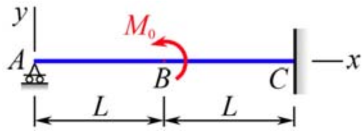


Fig. 6. Cantilever beam AC propped at A and carrying a moment at B

Solution. The beam is in static equilibrium. Its free-body diagram is shown in Fig. 7, where we note that the beam is statically indeterminate to the *first* degree.

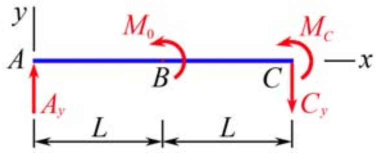


Fig. 7. Free-body diagram of the propped cantilever beam AC

● **Using MoI:** Applying the *method of integration* to this beam, we write

$$q = A_y < x >^{-1} - M_0 < x - L >^{-2}$$
$$V = A_y < x >^{0} - M_0 < x - L >^{-1}$$
$$M = EIy'' = A_y < x >^{1} - M_0 < x - L >^{0}$$
$$EIy' = \frac{A_y}{2} < x >^{2} - M_0 < x - L >^{1} + C_1$$
$$EIy = \frac{A_y}{6} < x >^{3} - \frac{M_0}{2} < x - L >^{2} + C_1 x + C_2$$

The *boundary conditions* of this beam reveal that $y(0) = 0$ at A, $y'(2L) = 0$ at C, and $y(2L) = 0$ at C. Imposing these *three* conditions, respectively, we write

$$0 = C_2$$
$$0 = \frac{A_y}{2}(2L)^2 - M_0 L + C_1$$
$$0 = \frac{A_y}{6}(2L)^3 - \frac{M_0}{2}L^2 + C_1(2L) + C_2$$

These *three* simultaneous equations yield

$$C_1 = -\frac{M_0 L}{8} \qquad C_2 = 0 \qquad A_y = \frac{9M_0}{16L}$$

Using these values and the foregoing equations for EIy' and EIy, we write

$$\theta_A = y'\big|_{x=0} = \frac{C_1}{EI} = -\frac{M_0 L}{8EI} \qquad \theta_B = y'\big|_{x=L} = \frac{A_y}{2EI}L^2 + \frac{C_1}{EI} = \frac{5M_0 L}{32EI}$$
$$y_B = y\big|_{x=L} = \frac{A_y}{6EI}L^3 + \frac{C_1}{EI}L = -\frac{M_0 L^2}{32EI}$$

We report that

$\mathbf{A}_y = \dfrac{9M_0}{16L}\uparrow$ $\theta_A = \dfrac{M_0 L}{8EI}\circlearrowright$ $\theta_B = \dfrac{5M_0 L}{32EI}\circlearrowleft$ $y_B = \dfrac{M_0 L^2}{32EI}\downarrow$

● **Using MoMF:** Let the *method of model formulas* be now applied to this beam. We note that this beam has a total length of $2L$, which will be the value for the *parameter* L in all the model formulas in Eqs. (1) through (4). We see that the deflection y_C and the slope θ_C at C, as well as the deflection y_A at A, are equal to zero. Applying the model formulas in Eqs. (3) and (4) to the entire beam, we write

$$0 = \theta_A + \frac{A_y(2L)^2}{2EI} + 0 - 0 + \frac{-M_0}{EI}(2L-L) - 0 - 0 + 0 + 0 + 0 - 0$$

$$0 = 0 + \theta_A(2L) + \frac{A_y(2L)^3}{6EI} + 0 - 0 + \frac{-M_0}{2EI}(2L-L)^2 - 0 - 0 + 0 + 0 + 0 - 0$$

These *two* simultaneous equations yield

$$A_y = \frac{9M_0}{16L} \qquad \theta_A = -\frac{M_0 L}{8EI}$$

Using these values and applying the model formulas in Eqs. (1) and (2), respectively, we write

$$\theta_B = y'\big|_{x=L} = \theta_A + \frac{A_y}{2EI}L^2 + 0 - 0 + 0 - 0 - 0 + 0 + 0 + 0 - 0 = \frac{5M_0 L}{32EI}$$

$$y_B = y\big|_{x=L} = 0 + \theta_A L + \frac{A_y}{6EI}L^3 + 0 - 0 + 0 - 0 - 0 + 0 + 0 + 0 - 0 = -\frac{M_0 L^2}{32EI}$$

We report that

$\mathbf{A}_y = \dfrac{9M_0}{16L}\uparrow$ $\theta_A = \dfrac{M_0 L}{8EI}\circlearrowright$ $\theta_B = \dfrac{5M_0 L}{32EI}\circlearrowleft$ $y_B = \dfrac{M_0 L^2}{32EI}\downarrow$

Example 4. A continuous beam AC with constant flexural rigidity EI and total length $2L$ has a roller support at A, a roller support at B, a fixed support at C and carries a linearly distributed load as shown in Fig. 8. Determine (a) the vertical reaction force $\mathbf{A}_y$ and slope θ_A at A, (b) the vertical reaction force $\mathbf{B}_y$ and slope θ_B at B.

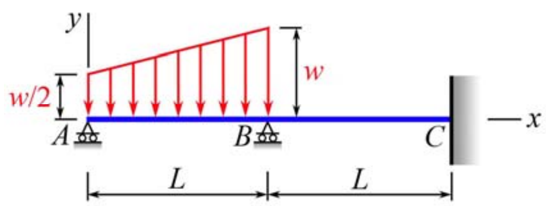


Fig. 8. Continuous beam AC carrying a linearly distributed load

Solution. The beam is in static equilibrium. Its free-body diagram is shown in Fig. 9, where we note that the beam is statically indeterminate to the *second* degree.

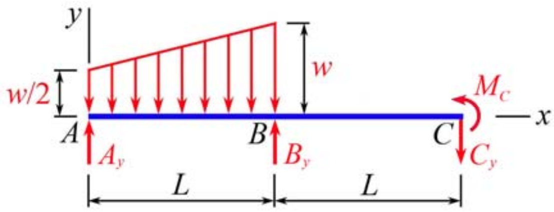


Fig. 9. Free-body diagram of the continuous beam *AC*

● **Using MoI:** Treating $\mathbf{B}_y$ as an applied unknown concentrated force, we may use *superposition* technique to first write the loading function *q* as follows:

$$q = A_y < x >^{-1} + B_y < x-L >^{-1} - \frac{w}{2} < x >^0 - \frac{w}{2L} < x >^1 + \frac{w}{2L} < x-L >^1 + w < x-L >^0$$

Applying the *method of integration* to this beam, we write

$$V = A_y < x >^0 + B_y < x-L >^0 - \frac{w}{2} < x >^1 - \frac{w}{4L} < x >^2 + \frac{w}{4L} < x-L >^2 + w < x-L >^1$$

$$M = EIy'' = A_y < x >^1 + B_y < x-L >^1 - \frac{w}{4} < x >^2 - \frac{w}{12L} < x >^3 + \frac{w}{12L} < x-L >^3 + \frac{w}{2} < x-L >^2$$

$$EIy' = \frac{A_y}{2} < x >^2 + \frac{B_y}{2} < x-L >^2 - \frac{w}{12} < x >^3 - \frac{w}{48L} < x >^4 + \frac{w}{48L} < x-L >^4 + \frac{w}{6} < x-L >^3 + C_1$$

$$EIy = \frac{A_y}{6} < x >^3 + \frac{B_y}{6} < x-L >^3 - \frac{w}{48} < x >^4 - \frac{w}{240L} < x >^5 + \frac{w}{240L} < x-L >^5 + \frac{w}{24} < x-L >^4 + C_1 x + C_2$$

The *boundary conditions* of this beam reveal that $y'(2L) = 0$ and $y(2L) = 0$ at *C*, $y(L) = 0$ at *B*, and $y(0) = 0$ at *A*. Imposing these *four* conditions, in *order*, we write

$$0 = \frac{A_y}{2}(2L)^2 + \frac{B_y}{2}L^2 - \frac{w}{12}(2L)^3 - \frac{w}{48L}(2L)^4 + \frac{w}{48L}L^4 + \frac{w}{6}L^3 + C_1$$

$$0 = \frac{A_y}{6}(2L)^3 + \frac{B_y}{6}L^3 - \frac{w}{48}(2L)^4 - \frac{w}{240L}(2L)^5 + \frac{w}{240L}L^5 + \frac{w}{24}L^4 + C_1(2L) + C_2$$

$$0 = \frac{A_y}{6}L^3 - \frac{w}{48}L^4 - \frac{w}{240L}L^5 + C_1 L + C_2$$

$$0 = C_2$$

The above *four* simultaneous equations yield

$$A_y = \frac{39wL}{140} \qquad B_y = \frac{31wL}{56} \qquad C_1 = -\frac{3wL^3}{140} \qquad C_2 = 0$$

Using these values and the foregoing equation for EIy', we write

$$\theta_A = y'\big|_{x=0} = \frac{C_1}{EI} = -\frac{3wL^3}{140EI} \qquad \theta_B = y'\big|_{x=L} = \frac{1}{EI}\left(\frac{A_y}{2}L^2 - \frac{w}{12}L^3 - \frac{w}{48L}L^4 + C_1\right) = \frac{23wL^3}{1680EI}$$

We report that

$$\boxed{\mathbf{A}_y = \frac{39wL}{140}\uparrow} \qquad \boxed{\theta_A = \frac{3wL^3}{140EI}\circlearrowright} \qquad \boxed{\mathbf{B}_y = \frac{31wL}{56}\uparrow} \qquad \boxed{\theta_B = \frac{23wL^3}{1680EI}\circlearrowleft}$$

● **Using MoMF:** Let the *method of model formulas* be now applied to this beam. We notice that the beam AC has a total length $2L$, which will be the value for the *parameter* L in all the model formulas in Eqs. (1) through (4). We see that the shear force V_A at left end A is equal to A_y, the moment M_A and deflection y_A at A are zero, the deflection y_B at B is zero, and the slope θ_C and deflection y_C at C are zero. Applying the model formulas in Eqs. (3) and (4) to the beam AC and using Eq. (2) to impose the condition that $y_B = y(L) = 0$ at B, in that *order*, we write

$$0 = \theta_A + \frac{A_y(2L)^2}{2EI} + 0 - \frac{-B_y}{2EI}(2L-L)^2 + 0 - \frac{w/2}{6EI}(2L)^3 - \frac{w-(w/2)}{24EIL}(2L)^4 + \frac{w}{6EI}(2L-L)^3 + \frac{w-(w/2)}{24EIL}(2L-L)^4 + 0 - 0$$

$$0 = 0 + \theta_A(2L) + \frac{A_y(2L)^3}{6EI} + 0 - \frac{-B_y}{6EI}(2L-L)^3 + 0 - \frac{w/2}{24EI}(2L)^4 - \frac{w-(w/2)}{120EIL}(2L)^5 + \frac{w}{24EI}(2L-L)^4 + \frac{w-(w/2)}{120EIL}(2L-L)^5 + 0 - 0$$

$$0 = 0 + \theta_A L + \frac{A_y}{6EI}L^3 + 0 - 0 + 0 - \frac{w/2}{24EI}L^4 - \frac{w-(w/2)}{120EIL}L^5 + 0 + 0 + 0 - 0$$

These *three* simultaneous equations yield

$$A_y = \frac{39wL}{140} \qquad \theta_A = -\frac{3wL^3}{140EI} \qquad B_y = \frac{31wL}{56}$$

Using these values and applying the model formula in Eq. (1), we write

$$\theta_B = y'|_{x=L} = \theta_A + \frac{A_y}{2EI}L^2 + 0 - 0 + 0 - \frac{w/2}{6EI}L^3 - \frac{w-(w/2)}{24EIL}L^4 + 0 + 0 + 0 - 0 = \frac{23wL^3}{1680EI}$$

We report that

$$\mathbf{A}_y = \frac{39wL}{140}\uparrow \qquad \theta_A = \frac{3wL^3}{140EI}\circlearrowright \qquad \mathbf{B}_y = \frac{31wL}{56}\uparrow \qquad \theta_B = \frac{23wL^3}{1680EI}\circlearrowleft$$

Example 5. A stepped beam AD, propped at A and fixed at D, carries a concentrated force $P\downarrow$ at B as shown in Fig. 10, where the segments AC and CD have flexural rigidities EI_1 and EI_2, respectively. Determine (*a*) the reaction force $\mathbf{A}_y$ at A; (*b*) the slopes θ_A, θ_B, and θ_C at A, B, and C; (*c*) the deflections y_B and y_C at B and C.

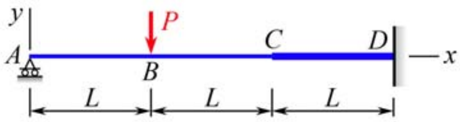


Fig. 10. Stepped beam AD being supported at A and D and loaded at B

Solution. The beam is in static equilibrium and is statically indeterminate to the *first* degree. To facilitate the analysis of this beam, we first draw the free-body diagrams of its segments AC and CD as shown in Fig. 11.

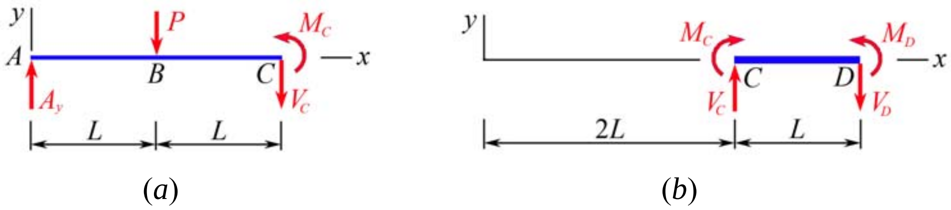


Fig. 11 Free-body diagrams of the segments *AC* and *CD*

● **Using MoI:** Applying the *method of integration* to the segment *AC* as shown in Fig. 11(*a*), we write

$$q_{AC} = A_y < x >^{-1} - P < x - L >^{-1}$$

$$V_{AC} = A_y < x >^{0} - P < x - L >^{0}$$

$$M_{AC} = EI_1 y''_{AC} = A_y < x >^{1} - P < x - L >^{1}$$

$$EI_1 y'_{AC} = \frac{A_y}{2} < x >^{2} - \frac{P}{2} < x - L >^{2} + C_1$$

$$EI_1 y_{AC} = \frac{A_y}{6} < x >^{3} - \frac{P}{6} < x - L >^{3} + C_1 x + C_2$$

Applying the *method of integration* to the segment *CD* as shown in Fig. 11(*b*), we write

$$q_{CD} = V_C < x - 2L >^{-1} + M_C < x - 2L >^{-2}$$

$$V_{CD} = V_C < x - 2L >^{0} + M_C < x - 2L >^{-1}$$

$$M_{CD} = EI_2 y''_{CD} = V_C < x - 2L >^{1} + M_C < x - 2L >^{0}$$

$$EI_2 y'_{CD} = \frac{V_C}{2} < x - 2L >^{2} + M_C < x - 2L >^{1} + C_3$$

$$EI_2 y_{CD} = \frac{V_C}{6} < x - 2L >^{3} + \frac{M_C}{2} < x - 2L >^{2} + C_3 x + C_4$$

The *boundary conditions* of the beam reveal that $y_{AC}(0) = 0$ at *A*; $y_{AC}(2L) = y_{CD}(2L)$ and $y'_{AC}(2L) = y'_{CD}(2L)$ at *C*; $y_{CD}(3L) = 0$ and $y'_{CD}(3L) = 0$ at *D*. Imposing these *five* conditions, in *order*, we write

$$0 = C_2 \tag{a}$$

$$\frac{1}{I_1}\left[\frac{A_y}{6}(2L)^3 - \frac{P}{6}(L)^3 + C_1(2L) + C_2\right] = \frac{1}{I_2}\left[C_3(2L) + C_4\right] \tag{b}$$

$$\frac{1}{I_1}\left[\frac{A_y}{2}(2L)^2 - \frac{P}{2}(L)^2 + C_1\right] = \frac{C_3}{I_2} \tag{c}$$

$$0 = \frac{V_C}{6}(L)^3 + \frac{M_C}{2}(L)^2 + C_3(3L) + C_4 \tag{d}$$

$$0 = \frac{V_C}{2}(L)^2 + M_C(L) + C_3 \tag{e}$$

For equilibrium of the segment *AC* in Fig. 11(*a*), we write

$$+\!\uparrow \Sigma F_y = 0: \qquad A_y - V_C - P = 0 \tag{f}$$

$$+\circlearrowleft \Sigma M_C = 0: \qquad -2LA_y + LP + M_C = 0 \tag{g}$$

The above *seven* simultaneous Eqs. (a) through (g) yield

$$A_y = \frac{(23I_1 + 5I_2)P}{2(19I_1 + 8I_2)} \qquad V_C = -\frac{(15I_1 + 11I_2)P}{2(19I_1 + 8I_2)} \qquad M_C = \frac{(4I_1 - 3I_2)PL}{19I_1 + 8I_2}$$

$$C_1 = -\frac{(I_1^2 + 31I_1I_2 + 4I_2^2)PL^2}{4I_2(19I_1 + 8I_2)} \qquad C_2 = 0$$

$$C_3 = \frac{(-I_1 + 23I_2)PL^2}{4(19I_1 + 8I_2)} \qquad C_4 = -\frac{(89I_2)PL^3}{6(19I_1 + 8I_2)}$$

Using these values and the foregoing equations for $EI_1 y'_{AC}$ and $EI_1 y_{AC}$ we write

$$\theta_A = y'_{AC}\big|_{x=0} = \frac{C_1}{EI_1} = -\frac{(I_1^2 + 31I_1I_2 + 4I_2^2)PL^2}{4EI_1I_2(19I_1 + 8I_2)}$$

$$\theta_B = y'_{AC}\big|_{x=L} = \frac{A_yL^2}{2EI_1} + \frac{C_1}{EI_1} = -\frac{(I_1^2 + 8I_1I_2 - I_2^2)PL^2}{4EI_1I_2(19I_1 + 8I_2)}$$

$$\theta_C = y'_{AC}\big|_{x=2L} = \frac{A_y(2L)^2}{2EI_1} - \frac{PL^2}{2EI_1} + \frac{C_1}{EI_1} = \frac{(-I_1 + 23I_2)PL^2}{4EI_2(19I_1 + 8I_2)}$$

$$y_B = y_{AC}\big|_{x=L} = \frac{A_yL^3}{6EI_1} + \frac{C_1L}{EI_1} = -\frac{(3I_1^2 + 70I_1I_2 + 7I_2^2)PL^3}{12EI_1I_2(19I_1 + 8I_2)}$$

$$y_C = y_{AC}\big|_{x=2L} = \frac{4A_yL^3}{3EI_1} - \frac{PL^3}{6EI_1} + \frac{2C_1L}{EI_1} = -\frac{(3I_1 + 20I_2)PL^3}{6EI_2(19I_1 + 8I_2)}$$

We report that

$$\boxed{\mathbf{A}_y = \frac{(23I_1 + 5I_2)P}{2(19I_1 + 8I_2)}\uparrow} \qquad \boxed{\theta_A = \frac{(I_1^2 + 31I_1I_2 + 4I_2^2)PL^2}{4EI_1I_2(19I_1 + 8I_2)}\circlearrowright}$$

$$\boxed{\theta_B = \frac{(I_1^2 + 8I_1I_2 - I_2^2)PL^2}{4EI_1I_2(19I_1 + 8I_2)}\circlearrowright} \qquad \boxed{\theta_C = \frac{(-I_1 + 23I_2)PL^2}{4EI_2(19I_1 + 8I_2)}\circlearrowleft}$$

$$\boxed{y_B = \frac{(3I_1^2 + 70I_1I_2 + 7I_2^2)PL^3}{12EI_1I_2(19I_1 + 8I_2)}\downarrow} \qquad \boxed{y_C = \frac{(3I_1 + 20I_2)PL^3}{6EI_2(19I_1 + 8I_2)}\downarrow}$$

● **Using MoMF:** Let the *method of model formulas* be now applied to this stepped beam. We first divide the beam into two segments, whose free-body diagrams are shown in parts (*a*) and (*b*) of Fig. 11. In particular, note that the segment *AC* has a total length 2*L*, which will be the value for the *parameter L* in all the model formulas in Eqs. (1) through (4). We see that shear force

V_A at the left end A is equal to A_y, the moment M_A and deflection y_A at A are zero. We may let the slope and deflection at C be θ_C and y_C, respectively. Applying the model formulas in Eqs. (3) and (4) to segment *AC* in Fig. 11(*a*), in that *order*, we write

$$\theta_C = \theta_A + \frac{A_y(2L)^2}{2EI_1} + 0 - \frac{P}{2EI_1}(2L-L)^2 + 0 - 0 - 0 + 0 + 0 + 0 - 0 \tag{h}$$

$$y_C = 0 + \theta_A(2L) + \frac{A_y(2L)^3}{6EI_1} + 0 - \frac{P}{6EI_1}(2L-L)^3 + 0 - 0 - 0 + 0 + 0 + 0 - 0 \tag{i}$$

For equilibrium of the segment *AC* in Fig. 11(*a*), we write

$$+\uparrow \Sigma F_y = 0: \qquad A_y - V_C - P = 0 \tag{j}$$

$$+\circlearrowleft \Sigma M_C = 0: \qquad -2LA_y + LP + M_C = 0 \tag{k}$$

We note that the slope θ_D and deflection y_D at end D of segment *CD* are zero. Applying the model formulas in Eqs. (3) and (4) to segment *CD* in Fig. 11(*b*), in that *order*, we write

$$0 = \theta_C + \frac{V_C L^2}{2EI_2} + \frac{M_C L}{EI_2} - 0 + 0 - 0 - 0 + 0 + 0 + 0 - 0 \tag{l}$$

$$0 = y_C + \theta_C L + \frac{V_C L^3}{6EI_2} + \frac{M_C L^2}{2EI_2} - 0 + 0 - 0 - 0 + 0 + 0 + 0 - 0 \tag{m}$$

The above *six* simultaneous Eqs. (h) through (m) yield

$$A_y = \frac{(23I_1 + 5I_2)P}{2(19I_1 + 8I_2)} \qquad V_C = -\frac{(15I_1 + 11I_2)P}{2(19I_1 + 8I_2)} \qquad M_C = \frac{(4I_1 - 3I_2)PL}{19I_1 + 8I_2}$$

$$\theta_A = -\frac{(I_1^2 + 31I_1I_2 + 4I_2^2)PL^2}{4EI_1I_2(19I_1 + 8I_2)} \qquad \theta_C = \frac{(-I_1 + 23I_2)PL^2}{4EI_2(19I_1 + 8I_2)} \qquad y_C = -\frac{(3I_1 + 20I_2)PL^3}{6EI_2(19I_1 + 8I_2)}$$

Using these values and applying the model formulas in Eqs. (1) and (2), we write

$$\theta_B = y'_{AC}\big|_{x=L} = \theta_A + \frac{A_y}{2EI_1}L^2 + 0 - 0 + 0 - 0 - 0 + 0 + 0 + 0 - 0 = -\frac{(I_1^2 + 8I_1I_2 - I_2^2)PL^2}{4EI_1I_2(19I_1 + 8I_2)}$$

$$y_B = y_{AC}\big|_{x=L} = 0 + \theta_A L + \frac{A_y}{6EI_1}L^3 + 0 - 0 + 0 - 0 - 0 + 0 + 0 + 0 - 0 = -\frac{(3I_1^2 + 70I_1I_2 + 7I_2^2)PL^3}{12EI_1I_2(19I_1 + 8I_2)}$$

We report that

$$\boxed{\mathbf{A}_y = \frac{(23I_1 + 5I_2)P}{2(19I_1 + 8I_2)}\uparrow} \qquad \boxed{\theta_A = \frac{(I_1^2 + 31I_1I_2 + 4I_2^2)PL^2}{4EI_1I_2(19I_1 + 8I_2)}\circlearrowright}$$

$$\boxed{\theta_B = \frac{(I_1^2 + 8I_1I_2 - I_2^2)PL^2}{4EI_1I_2(19I_1 + 8I_2)}\circlearrowright} \qquad \boxed{\theta_C = \frac{(-I_1 + 23I_2)PL^2}{4EI_2(19I_1 + 8I_2)}\circlearrowleft}$$

$$\boxed{y_B = \frac{(3I_1^2 + 70I_1I_2 + 7I_2^2)PL^3}{12EI_1I_2(19I_1 + 8I_2)}\downarrow} \qquad \boxed{y_C = \frac{(3I_1 + 20I_2)PL^3}{6EI_2(19I_1 + 8I_2)}\downarrow}$$

● **Checking Obtained Results:** The effort to obtain the solution for the problem in this example is algebraically challenging. Naturally, it is desirable to check the preceding obtained results against a known solution for the special case of

$$I_1 = I_2 = I$$

For such a special case, the preceding obtained results degenerate into the following:

$$\mathbf{A}_y = \frac{14P}{27}\uparrow \qquad \theta_A = \frac{PL^2}{3EI}\circlearrowright \qquad \theta_B = \frac{2PL^2}{27EI}\circlearrowright$$

$$\theta_C = \frac{11PL^2}{54EI}\circlearrowleft \qquad y_B = \frac{20PL^3}{81EI}\downarrow \qquad y_C = \frac{23PL^3}{162EI}\downarrow$$

We find that these special results are indeed *consistent* with those given at the end of textbooks.[9]

III. An Effective Approach to Teaching the MoMF

The *method of model formulas* is a general methodology that employs a set of *four equations* to serve as *model formulas* in solving problems involving statically indeterminate reactions, as well as slopes and deflections, of elastic beams. The first two model formulas are for the slope and deflection at any position *x* of the beam and contain rudimentary singularity functions, while the other two model formulas contain only traditional algebraic expressions. Generally, this method requires much less effort in solving beam deflection problems. Most students favor this method because they can solve problems in shorter time using this method and they score higher in tests.

The five examples, arranged in order of increasing challenge, in Section II provide a variety of head-to-head comparisons between solutions by the traditional *method of integration* and those by the *method of model formulas*; and all of the solutions are, respectively, in agreement. Thus, all solutions by the *method of model formulas* are naturally correct. Experience shows that the following steps form a pedagogy that can be used to effectively introduce and teach the *method of model formulas* to students to enrich their study and set of skills in determining statically indeterminate reactions and deflections of elastic beams in mechanics of materials:

- Teach the traditional *method of integration* and the imposition of boundary conditions.
- Teach the rudiments of *singularity functions* and utilize them in the *method of integration*.
- Go over briefly the derivation[1] of the *four model formulas* in terms of *singularity functions*.
- Give students the heads-up on the following advantages in the *method of model formulas*:
  - No need to integrate or evaluate constants of integration.
  - Not prone to generate a large number of simultaneous equations **even if**
    - ▷ the beam carries multiple concentrated loads (forces or moments),
    - ▷ the beam has one or more simple supports *not* at its ends,
    - ▷ the beam has linearly distributed loads *not* starting at its left end, and
    - ▷ the beam has linearly distributed loads *not* ending at its right end.
- Demonstrate solutions of several beam problems by the *method of model formulas*.
- Assess the solutions obtained (e.g., comparing with solutions by another method).

Although solutions obtained by the *method of model formulas* are often more direct than those obtained by the *method of integration*, a **one-page excerpt** from the *method of model formulas*, such as that shown in Fig. 1, must be made available to those who used this method. Still, one may remember that a **list of formulas** for slope and deflection of selected beams having a variety of supports and loading is *also* needed by persons who use the method of superposition. In this regard, the *method of model formulas* is **on a par with** the *method of superposition*.

IV. Concluding Remarks

In the *method of model formulas*, no explicit integration or differentiation is involved in applying any of the model formulas. The model formulas essentially serve to provide *material equations* (which involve and reflect the material property) besides the equations of static equilibrium of the beam that can readily be written. Selected applied loads are illustrated in Fig. 1(*a*), which cover most of the loads encountered in undergraduate Mechanics of Materials. In the case of a nonlinearly distributed load on the beam, the model formulas may be modified by the user for such a load.

The *method of model formulas* is best taught to students as an alternative method, after they have learned one or more of the traditional methods.[2-12] This new method enriches students' study and set of skills in determining reactions and deflections of beams, and it provides engineers with a means to independently check their solutions obtained using traditional methods.

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ING-CHANG JONG
Ing-Chang Jong is Professor of Mechanical Engineering at the University of Arkansas. He received his BSCE in 1961 from the National Taiwan University, his MSCE in 1963 from South Dakota School of Mines and Technology, and his Ph.D. in Theoretical and Applied Mechanics in 1965 from Northwestern University. Dr. Jong was Chair of the Mechanics Division, ASEE, in 1996-97. He received the Archie Higdon Distinguished Educator Award in 2009.

WILLIAM T. SPRINGER
William T. Springer is Associate Professor of Mechanical Engineering at the University of Arkansas. He received his BSME in 1974 from the University of Texas at Arlington, his MSME in 1979 from the University of Texas at Arlington, and his Ph.D. in Mechanical Engineering in 1982 from the University of Texas at Arlington. Dr. Springer is active in ASME where he received the Dedicated Service Award in 2006 and attained Fellow Grade in 2008.

RICK J. COUVILLION
Rick J. Couvillion is Associate Professor of Mechanical Engineering at the University of Arkansas. He received his BSME in 1975 from the University of Arkansas. After 2½ years in industry, he returned to school, received his MSME from Georgia Tech in 1978, and his Ph.D. from Georgia Tech in 1981. He is active in ASHRAE and was elected an ASME fellow in 2004. He was chosen for the University of Arkansas Teaching Academy in 2005.